Homework 6

Section 6.1

6. There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?

Solution:

By the product rule: \(4 \times 6\)

28. How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

Solution: \(10^3 \times 26^3 + 26^3 \times 10^3\)

Three digits followed by three uppercase letters (Product rule)
Or (Sum rule)
Three uppercase letters followed by three digits (Product rule)

32. How many strings of eight uppercase English letters are there
   a) if letters can be repeated?
   b) if no letter can be repeated?
   c) that start with X, if letters can be repeated?
   d) that start with X, if no letter can be repeated?
   e) that start and end with X, if letters can be repeated?
   f) that start with the letters BO (in that order), if letters can be repeated?
   g) that start and end with the letters BO (in that order), if letters can be repeated?
   h) that start or end with the letters BO (in that order), if letters can be repeated?

Solution:
a) \(26^8\)
b) \(26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19\)
c) \(26^7\)
d) \(1 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19\)
e) \(26^6\)
f) \(26^6\)
g) \(26^4\)
h) \(26^6 + 26^6 - 26^4\)
Section 6.2

2. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

Solution:

Number of letters: 26

Follows from the pigeonhole principle: \( \lceil \frac{30}{26} \rceil \)

4. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
   a) How many balls must she select to be sure of having at least three balls of the same color?
   b) How many balls must she select to be sure of having at least three blue balls?

Solution:

a) There are two colors: these are the pigeonholes. By the generalized pigeonhole principle, if \( N \) balls are selected, at least \( \lceil N/2 \rceil \) must have the same color. \( N \geq 2 \times 2 + 1 = 5 \). The answer is 5.

b) There are 10 red balls. So if we select 10 + 3 balls, it is insured that at least three blue balls.
Section 6.3

18. A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes are there in total?
   a) contain exactly three heads?
   b) contain at least three heads?
   c) contain the same number of heads and tails?

Solution
   a) \(2^8\)
   b) \(C(8,3)\)
   c) \(C(8,3) + C(8,4) + C(8,5) + C(8,6) + C(8,7) + C(8,8)\)
   d) \(C(8,4)\)

26. Thirteen people on a softball team show up for a game.
   a) How many ways are there to choose 10 players to take the field?
   b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
   c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

Solution
   a) \(C(13,10)\)
   b) \(P(13,10)\)
   c) \(C(13,10) - C(10,10)\)
Section 6.4

2. In how many different ways can five elements be selected in order from a set with five elements when repetition is allowed?

Solution: $5^5$

4. Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

Solution: $6^7$

6. How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?

Solution:

\[ C(3+5-1, 5) = C(7,5) = C(7,2) \]