A Study of the Routing and Spectrum Allocation in Spectrum-sliced Elastic Optical Path Networks

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Abstract—In OFDM-based optical networks, multiple sub-carriers can be allocated to accommodate various size of traffic demands. By using the multi-carrier modulation technique, sub-carriers for the same node-pair can be overlapping in the spectrum domain. Compared to the traditional wavelength routed networks (WRNs), the OFDM-based Spectrum-sliced Elastic Optical Path (SLICE) network has higher spectrum efficiency due to its finer granularity and frequency-resource saving. In this work, for the first time, we comprehensively study the routing and spectrum allocation (RSA) problem in the SLICE network. After proving the NP-hardness of the static RSA problem, we formulate the RSA problem using the Integer Linear Programming (ILP) formulations to optimally minimize the maximum number of sub-carriers required on any fiber of a SLICE network. We then analyze the lower/upper bounds for the sub-carrier number in a network with general or specific topology. We also propose two efficient algorithms, namely, balanced load spectrum allocation (BLSA) algorithm and shortest path with maximum spectrum reuse (SPSR) algorithm to minimize the required sub-carrier number in a SLICE network. The results show that the proposed algorithms can match the analysis and approximate the optimal solutions using the ILP model.

I. Introduction

Traditional WDM networks adopt a fixed-size frequency allocation per wavelength, which is the smallest granularity to accommodate the traffic demands [1], [2]. Each wavelength in WDM networks is separated from other wavelengths by guard-band frequencies to ensure signal quality and filtering at the receivers. The process of assigning wavelengths and routing wavelengths through intermediate nodes as lightpaths to satisfy the traffic demands is defined as the routing and wavelength assignment (RWA) problem, and the resulted network is referred to as the wavelength routed network (WRN) [1], [2]. The fixed-size frequency allocation in a WRN, however, has drawbacks in its coarse granularity and less flexibility. For example, when the traffic demand (even after grooming) is less than the capacity of one wavelength, an entire wavelength may still be allocated to accommodate this demand in WRNs. Moreover, when one traffic demand requires multiple wavelengths, it is not possible to eliminate the spectral gap (i.e., the guard-band frequencies to facilitate the filtering) between any two wavelengths. To address such issues and efficiently utilize the available spectrum in a fiber, the concept of the Spectrum-sliced Elastic Optical Path (SLICE) network is recently proposed [3]–[5].

One prominent characteristic of the SLICE network is the signal modulation technique of Orthogonal Frequency Division Multiplexing (OFDM), which can overcome the impairments due to the chromatic dispersion (CD) and polarization mode dispersion (PMD) in high-capacity long-haul fiber transmission [4]. Instead of using a single carrier, OFDM distributes the data on a bunch of sub-carriers, which can be partially overlapping in the spectrum domain. Each sub-carrier has a lower data rate and finer granularity than a single wavelength in WDM networks, thus can efficiently accommodate sub-wavelength traffic demands. When one traffic demand requires multiple sub-carriers, consecutive sub-carriers can be allocated and overlapping in the spectrum domain without using the spectral gap or guard-band frequencies. As shown in [3]–[5], SLICE networks can bring multiple benefits: (i) OFDM enables the expansion and contraction of the sub-carriers along an optical path, thus elastic bandwidth sharing can be achieved by adjusting the spectral bandwidth based on various users’ time-dependent complementary demands. (ii) It can facilitate energy efficient network operation. For example, the transponders can partially shut down the electronic drivers by reducing the number of OFDM sub-carriers to save energy. (iii) It can provide the capability of bandwidth-squeezed restoration by squeezing the bandwidth of the failed working optical path to ensure the minimum connectivity and transit through an alternative path, even though the alternative path may not have equivalent bandwidth as the original path. (iv) It can alleviate the reachability and generator placement issues in heterogeneous WDM networks [6].

In SLICE networks, a spectrum path is an all-optical trail established between the source and sink nodes by using one or multiple consecutive sub-carriers. The fundamental problem in the SLICE network is to route and allocate spectrum resources (or set up spectrum paths) to accommodate the traffic demands, which is defined as the routing and spectrum allocation (RSA) problem [3]. Note that the RSA problem is different from and more challenging than the traditional RWA problem. First, the RSA problem has to consider the routing and constraints such as the continuous availability of the sub-carriers, and the capacity of each sub-carrier on each fiber. Moreover, OFDM technology requires that for a
given spectrum path, the allocated sub-carriers have to be consecutive in spectrum domain to be effectively modulated [5], which complicates the routing and spectrum allocation process. Hereafter we refer to this requirement as the sub-carrier consecutiveness constraint. Second, the sub-carriers of the same spectrum path can be consecutive and overlapping in the spectrum domain. However, physical frequency filtering requires that various spectrum paths are separated in the spectrum domain by guard frequencies when two spectrum paths share one or more common fiber links. These guard frequencies are referred to as guard-carriers in this paper. Unlike the WDM network where guard-band frequencies are pre-allocated and fixed, the guard-carriers in SLICE network can be any of the sub-carriers and are determined in the process of spectrum paths establishment. The existence of the guard-carrier constraint further complicates the spectrum allocation. In the literature, the enabling technologies on hardware design of SLICE networks were discussed in [3]–[5] and the reference therein. In [4], the authors discussed the design of SLICE transceivers and switching nodes for the SLICE network. In [3], the enabling technologies of SLICE network and the feasibility of the SLICE network as a solution for the optical network of next decades were investigated. In [5], the authors studied the filtering characteristic of SLICE networks, and the experiments showed that the SLICE network can outperform the WDM network by a dramatic scale in terms of the spectrum efficiency. However, none of above work studied the detailed algorithms and complexity issues of the routing and spectrum allocation in SLICE networks.

In this paper, we study the routing and the spectrum allocation problem in SLICE networks with the goal of efficiently utilizing the spectrum resources or sub-carriers. We assume the off-line or static traffic pattern is applied in the SLICE network of this study. To the best of our knowledge, this is the first comprehensive study on the RSA problem in SLICE networks. The major contributions of this work are: (i) We formally define the routing and spectrum allocation (RSA) problem and prove its NP-hardness. (ii) We present Integer Linear Programming (ILP) formulations for the static RSA problem, which can optimally allocate the sub-carriers and guard-carriers in the network. (iii) We analyze the lower/upper bounds of the required sub-carriers in networks with different topologies. (iv) Efficient heuristic algorithms, namely balanced load spectrum allocation (BLSA) and shortest path with maximum spectrum reuse (SPSR), are proposed to solve the RSA problem in a large network, which are shown to be near-optimal in the simulation studies. As one of the pioneering work in this new field, we expect this paper can trigger more studies including, for example, the protection/restoration issues, the dynamic RSA problem (with fluctuate/incremental traffic) of the SLICE network in the future.

The rest of the paper is organized as follows. In Section II, we discuss the important concepts and the node architecture in the SLICE network. Then we investigate the complexity of the RSA problem in Section III. In Section IV, we formulate the RSA problem using the ILP formulations. In Section V, we present the lower/upper bounds analysis on the maximum sub-carrier number of any fiber in the SLICE network. In Section VI, we propose heuristic algorithms for the RSA problem. Finally, we present the numerical analysis of the proposed schemes in Section VII and conclusions in Section VIII.

II. OFDM, Sub-carrier, and SLICE Node Architecture

In the SLICE network, the finer granularity and elastic right-size bandwidth allocation is achieved with the aid of optical orthogonal frequency-division multiplexing (OFDM). In the frequency domain, one sub-carrier normally corresponds to several GHz, and the capacity of one sub-carrier is in the order of Gbps. OFDM enables both the sub-wavelength and super-wavelength accommodation in the SLICE network. Specifically, sub-wavelength accommodation can be achieved in the optical domain since a single sub-carrier that has a lower data rate than one wavelength of WDM networks. For super-wavelength traffic demands, spectrum paths can be created by assigning multiple consecutive sub-carriers, which can be overlapped in frequency domain at the OFDM transponders [4]. For a given traffic demand, the request can be translated into the number of sub-carriers by considering configurations such as the modulation level and OFDM implementation [4], 1, and accommodated through the establishment of the corresponding spectrum path. To form the spectrum path for the traffic demand using multiple sub-carriers, the SLICE network needs to deploy bandwidth-variable (BV) transponders at the network edge and bandwidth-variable WXC s in the network core, which can be built based on the continuous bandwidth-variable wavelength-selective switch (WSS) [4], [8], [9]. Note that two spectrum paths that share one or more common fiber links, have to be separated in frequency domain to facilitate optical signal filtering. In other words, two sets of sub-carriers, within the two spectrum paths have to be isolated by a guard-carrier. The size of the guard-carrier, however, is not trivial and may be in the order of one or multiple sub-carrier(s) [5]. In the following discussions, we assume that the guard-carrier required to separate two spectrum paths consists of GC sub-carriers. One example of routing the spectrum paths using the WXC node in a SLICE network is shown in Fig. 1, where Fig. 1(a) is a star network with 2 directional fibers per link and GC = 1. The BV WSSs are arranged with a broadcast-and-select configuration. The local traffic can be added and dropped through the connection to the OFDM transmitter and receiver, respectively. In the network shown in Fig. 1(a), there is a spectrum path $SP_1$ of 2 sub-carriers from $A$ to $B$, and there is another spectrum path $SP_2$ of 1 sub-carrier from $A$ to $C$. Figure 1(b) shows the spectrum allocation on Fiber $F_1$ for $SP_1$ and $SP_2$. As shown in Fig. 1(b), each sub-carrier on the fiber has an index. The sub-carrier with index 1 and 2 are assigned to $SP_1$ which requires 2 consecutive sub-carriers. The sub-carrier with index 4 is assigned to $SP_2$. The sub-carrier

1At the time of preparing the final version of this paper, efforts have been made on the Frequency Slot standardization of SLICE networks [7]. Note that approaches hereafter can also be adapted to accommodate requests based on the Frequency Slot concept (instead of sub-carriers).
with index 3 is assigned as the guard-carrier between \(SP_1\) and \(SP_2\) since they are overlapping on Fiber \(F_1\). Since the sub-carriers within \(SP_1\) are consecutive and no guard frequency (i.e., guard-carrier) is needed. However, to be able to separate signal \(SP_1\) from \(SP_2\) on Fiber \(F_1\), guard-carrier is necessary. Hence, we cannot use the sub-carrier 3 for \(SP_2\). As a result, to accommodate \(SP_1\) and \(SP_2\), Fiber \(F_1\) requires 4 sub-carriers. Clearly, the required sub-carrier number on Fiber \(F_1\) depends on the employed sub-carrier with the maximum index denoted by \(M_f\). We use \(MS = \max_{\nu} M_f\) to represent the maximum index of the sub-carriers allocated among all the fibers in a SLICE network. Hence, if there are no other traffic demands in Fig. 1(b), \(MS\) of the network will be 4. Note that Fig. 1(c) shows the switching configuration at Node \(N\). The traffic from \(A\) to \(N\) (through Fiber \(F_1\)) is sent to BV WSSs 2 and 3 to filter out to the Node \(B\) or \(C\).

### III. Routing and Spectrum Allocation (RSA)

In this section, we discuss the complexity of the static routing and spectrum allocation (RSA) problem. The RSA problem contains both the routing decision for all the node-pairs with non-zero traffic demands, and the sub-carrier allocation to satisfy the corresponding traffic demands. If the routing is already known or predetermined, the RSA problem turns out to be the static spectrum allocation (SRA) problem. In the following, we show the NP-Completeness of the RSA problem via its connection with the static lightpath establishment (SLE) problem in WRNs. Therefore the optimal RSA problem which jointly optimizes the routing and spectrum allocation is NP-hard. As to be shown in Section IV, the objective of the optimal RSA problem in this study is to minimize the maximum sub-carrier number required in any fiber of a SLICE network.

#### A. SLE problem in WRNs

Define a network as \(G(V, E, W)\) where \(V\) represents the set of \(N\) nodes, \(E\) represents the set of directional fiber links between nodes in \(V\), and \(W\) is the set of wavelengths on each fiber, \(|W| = \omega\). To establish one lightpath, we need one continuous available wavelength along its path.

**Definition:** Static Lightpath Establishment (SLE) problem - given a network \(G(V, E, W)\), \(\omega \geq 3\), and a predefined set of lightpaths \(LS\), is it possible to establish all lightpaths in the set [10]?

The SLE problem is proven to be NP-Complete in [10] which employs a reduction to (and from) the \(n\)-graph-colorability problem.

#### B. SRA problem in the SLICE network

Define a network as \(G(V, E, P)\), where \(V\) represents the set of nodes, \(E\) represents the set of directional fiber links between nodes in \(V\), and \(P\) is the set of sub-carriers on each fiber, \(|P| = \phi\).

**Definition:** Static Spectrum Allocation (SRA) problem - given a network \(G(V, E, P)\), and a predefined set of spectrum paths request pair \(SP \equiv \{<p_i, t_i>\}\), where \(p_i\) is the path and \(t_i\) is the request size (in terms of the number of sub-carriers) of the \(i\)-th spectrum path, is it possible to establish each spectrum path in the set using consecutive sub-carriers, and satisfy the guard-carrier constraint?

**Theorem 1:** The SRA problem is NP-Complete.

In the following, we sketch the proof of Theorem 1. (i) \(SRA \in NP\). The certificate \(C\) is the spectrum allocation \(<b_i, e_i>\) for each spectrum path \(p_i\), where \(b_i\) and \(e_i\) are the starting and ending sub-carrier index, respectively. The verifier: \(VF = \"On input C\"\): (1) Test whether each \(<b_i, e_i>\) satisfies the traffic demand \(t_i\) and the index is no greater than \(\phi\). (2) For each spectrum path \(p_i\), test whether it is separated by GC sub-carriers from any other \(p_j\) when they share a common fiber link. (3) If both pass, accept; otherwise, reject. The verifier runs in time \(O(|SP| + |SP|^2)\), which is polynomial in the size of the problem. (ii) The SLE problem is reducible to the SRA problem. Give a SLE problem on the network \(G(V, E, W)\) and lightpath set \(LS\), we can reduce it to a SRA problem using the following construction. We construct the SRA problem on a network \(G(V, E, P)\) with \(|W| = |P|\) and a set of spectrum paths \(SP\). For any lightpath request in \(LS\), if it requires 1 wavelength along path \(p_i\), it has a request of 1 sub-carrier along the path \(p_i\) in the SRA problem while assuming the guard-carrier size \(GC = 0\). (iii) It is easy to see that the reduction is in poly-nominal time of the problem size. (iv) The SLE problem has a solution if and only if the constructed SRA problem has a solution.

Through the above reduction, we can see that the spectrum allocation problem alone is hard. The RSA problem is even more challenging since the routing decision also has to be jointly considered. In the following sections, we study the RSA problem using Integer Linear Programming and lower/upper bounds analysis techniques as well as heuristic algorithms.

#### IV. ILP Model for the Optimal RSA

In this section, we develop formulations to model the optimal RSA problem using the Integer Linear Programming (ILP) technique.

**A. Notations and Variables**

- \(\phi\): The number of sub-carriers on a fiber;
- \(I_n\): The set of nodes connected to Node \(n\) by incoming fibers to \(n\);
- \(O_n\): The set of nodes connected to Node \(n\) by outgoing fibers from \(n\);
- \(\Xi_{n,m}\): Traffic demands matrix; the element \(T_{n,m}\) represents the traffic demands between Node \(n\) and Node \(m\) in terms of number of sub-carriers;
- \(GC\): The size of a guard-carrier in terms of the number of sub-carriers;
- \(V_{w,i,o,s,d}\): 1, if there is a spectrum path using sub-carrier \(w\) to satisfy the traffic demand between node-pair \((s,d)\) going from Node \(i\) to Node \(o\) and 0 otherwise;
- \(MS\): The maximum index of the sub-carriers allocated among all the fibers in the network.

**Minimize** \(MS\) \hspace{1cm} (1)
B. Objective of the RSA problem

Since more sub-carriers on a fiber imply more cost on the fiber, power consumption and corresponding switching equipments, the objective in this study is to minimize the maximum sub-carrier index among all the fibers, which is shown in Eq. (1). Meanwhile, we need Eq. (2) to obtain the maximum index of the sub-carriers among all the fiber links.

C. Constraints

1) Traffic Demand Constraint: Equations (3-4) specify that the traffic demands for node-pair \((s, d)\) should be exactly added at Node \(s\) and dropped at Node \(d\). Equation (5) makes sure that no traffic is added and dropped at the same node.

2) Sub-carrier Capacity Constraint: Equation (6) guarantees that one sub-carrier can only be used for satisfying one spectrum path.

3) Spectrum Continuity Constraint: The spectrum continuity constraint specifies that the spectrum path should use the same spectrum(s) along its routing path, which is shown in Eq. (7).

4) Guard-Carrier Constraint: When two spectrum paths are overlapping in terms of their routing path, the corresponding allocated spectrum slices have to be separated by a guard-carrier of \(GC\) sub-carriers. Thus, if \(V^w_{i,o,s,d} = 1\) for some \(w\) on Fiber \(i\)-\(a\), then all the sub-carriers within \([w-GC, w+GC]\) cannot be used for any other node-pairs’ spectrum paths. To model above if-then relationship using ILP, we introduce a large number \(B\), and use Eq. (8) to represent the constraint. Clearly, if \(V^w_{i,o,s,d}\) equals 1, then it exactly is the above if-then relationship. On the other hand, if the considered sub-carrier \(w\) are not used for node-pair \((s, d)\), this constraint is virtually omitted from the ILP model since the left side of Eq. (8) is small enough (\(B\) dominates) to make the Eq. (8) a tautology. The same technique is used in Eq. (9) and Eq. (10).

V. Lower/Upper Bounds Analysis for the Sub-carrier Number in SLICE Networks

In this section, we analyze the lower/upper bounds for the maximum sub-carrier index (i.e., \(MS\)) within the SLICE network. We assume that for a network with \(N\) nodes (and \(|E|\) edges), there are 2 unidirectional fibers per link and uniform traffic demands \(X\) between each node-pair. In the case with predetermined routing, this study focuses on the bound
analysis of ring networks while the analysis on the case with predetermined routing stands for general mesh networks.

A. MS without Predetermined Routing

Ring topology is widely adopted in the optical network due to its sparse link connection and inherent robustness under any single link failure. We hereafter analyze the lower/upper bound of MS for a ring network.

We use the cut-set (CS) technique [11]–[13] to analyze the lower bound of MS. A cut separates the network with N nodes into 2 disjoint induced sub-graphs. All the traffic demands between these 2 disjoint sub-graphs are carried by the links that compose the cut. If we assume the two sub-graphs contains S and N – S nodes, respectively, the traffic demands carried by the cut U are 2 * S * (N – S) * X sub-carriers. Since there are 2 * S * (N – S) various node-pairs, we need 2 * S * (N – S) * GC sub-carriers as the guard-carriers. The sub-carrier number required on one fiber is the ratio between the traffic demands carried by the cut U and the number of fibers in the cut (i.e., 2 * |U| due to 2 unidirectional fibers per link) since the maximal sub-carrier number on a fiber of the cut U is minimized (i.e., the lower bound) when traffic demands are evenly distributed among all the fibers of cut U. Moreover, for the spectrum path that has the largest sub-carrier index, it does not need a guard-carrier above the spectrum path with the highest index. Thus we can reduce GC sub-carriers from the above ratio, which finally produces the lower bound as in Eq. (11) 2.

$$MS \geq \left(\max_{V \in \text{cut}} \frac{S \cdot (N - S)}{|U|}\right) \cdot (X + GC) - GC$$

(11)

1) Lower Bound: We analyze the lower bound of MS in a ring network for the case with even and odd number of nodes, respectively. In a ring network, a cut contains 2 links, and the cut that yields the lower bound is the one that divides the ring nodes as equally as possible. Consequently, for a ring network with N nodes and N is even, both cuts contain \(\frac{N}{2}\) nodes. Similarly, the two cuts in a ring network with odd number of nodes have \(\frac{N+1}{2}\) and \(\frac{N-1}{2}\) nodes, respectively. The cut that produces the lower bound are shown in Fig. 2(a) and Fig. 2(b) with the dotted line. Consequently, we can obtain the lower bound in Theorem 2.a and 2.b.

Theorem 2.a: If N is even in a ring network, then MS \(\geq (X + GC) \cdot \left[\frac{N^2}{8}\right] - GC\).

Theorem 2.b: If N is odd in a ring network, then MS \(\geq (X + GC) \cdot \left[\frac{N^2-1}{8}\right] - GC\).

2) Upper Bound: We note that for a ring network, the upper bound of sub-carrier number equals to the lower bound, as indicated in Theorem 3.

Theorem 3: The lower bound and the upper bound on MS are tight in a ring network with uniform traffic demands.

Proof: Using induction, we prove Theorem 3 by showing that employing the shortest path routing and a specific spectrum allocation can achieve the lower bound in the ring network 3. The proof includes 2 cases as follows.

Case 1 Ring with even number of nodes: To simplify the proof, we first assume \(X = 1\) and \(GC = 0\). As the basis, Figure 3 shows that \(\left[\frac{N^2}{8}\right]\) sub-carrier is enough for the case \(N = 2\), and \(\left[\frac{N^2}{8}\right]=2\) sub-carriers are sufficient for the case \(N = 4\). For the node-pair \((1, 3)\) and \((2, 4)\) in Fig. 3(b) that have the maximum distance \(\frac{N}{2}\), we assign one sub-carrier along the clockwise ring and one along counterclockwise ring to carry the traffic. There are \(\frac{N}{2}\) node-pairs with the maximum distance in a ring network. We distribute them evenly on the clockwise and counterclockwise rings to minimize the maximum sub-carrier number of a fiber. For the remaining spectrum paths (with less than \(\frac{N}{2}\) hops), we only show the connection for one direction (say clockwise) in Fig. 3. We use the fibers along the opposite ring (say counterclockwise) for the connection of the other direction. Now we assume that \(\left[\frac{N^2}{8}\right]\) sub-carriers are sufficient for any ring with \(N\) nodes. As shown in Fig. 4, we add 2 extra nodes (Node \(N + 1\) and \(N + 2\)) diametrically opposite to each other. The extra traffic introduced includes the traffic between Node \(N + 1\), \(N + 2\) to the left-half and right-half original \(\frac{N}{2}\) nodes, as well as the traffic between Node \(N + 1\) and \(N + 2\). For the former part, we note that only \(\frac{N}{2}\) sub-carriers are necessary. This is because the sub-carriers used from Node \(N + 1\) (or \(N + 2\)) to the left-half can be reused from Node \(N + 1\) (or \(N + 2\)) to the right-half nodes. Moreover, for the left (or right) half only, the same sub-carriers can be reused for one node to both Node \(N + 1\) and \(N + 2\). For example, sub-carrier 1 (in red) can be used from Node \(N + 1\) to Node 1, then reused between Node 1 and Node \(N + 2\). Thus we can conclude that the extra sub-carriers required due to the traffic from \(N + 1\) and \(N + 2\) to original \(\frac{N}{2}\) nodes are \(\frac{N}{2}\). For the traffic between Node \(N + 1\) and \(N + 2\) (with maximum distance), however, whether it causes extra sub-carriers or not depends on the parity of \(\frac{N}{2}\). Accordingly, we further separate the proof into two scenarios:

3Note that two fibers per link create a clockwise and a counterclockwise ring.

Fig. 2. A ring network with \(N\) nodes

\[\text{Fig. 2. A ring network with } N \text{ nodes}\]
Case 1.1: When \( N = 4 \times k \) for some integer \( k(>0) \), we have \( \left\lceil \frac{N^2}{8} \right\rceil = \frac{N^2}{8} \), and the basis is \( N = 4 \). After adding 4 nodes to this ring, the sub-carrier increase for the traffic between original nodes and the new 4 nodes is \( \frac{N^2}{2} + \frac{N^2}{2} = N + 1 \). In addition, we need 1 extra sub-carrier for the traffic from \( N + 1 \) to \( N + 2 \) and \( N + 3 \) to \( N + 4 \). Hence the additional number of sub-carriers is \( N + 1 + 1 = N + 2 \). The number of sub-carrier required for ring with \( N + 4 \) nodes consequently is \( \frac{N^2}{2} + N + 2 = \left( \frac{N+4}{2} \right)^2 \).

Case 1.2: When \( N = 4 \times k + 2 \) for some integer \( k \), we have \( \left\lceil \frac{N^2}{8} \right\rceil = 2 \times k^2 + 2 \times k + 1 \) and the basis is \( N = 2 \). Adding 4 nodes increases the sub-carrier number by \( N + 2 = 4 \times k + 4 \). Thus the sub-carrier number required for ring with \( N + 4 \) nodes is \( \left\lceil \frac{N^2}{8} \right\rceil + N + 2 = 2 \times k^2 + 6 \times k + 5 = \left( \frac{N+4}{2} \right)^2 \).

Case 2 Ring with odd number of nodes: As the basis shown in Fig. 3, \( \frac{N^2-1}{8} = 1 \) sub-carrier is enough for the case \( N = 3 \). Assume the bound is tight for any \( N \), and we add 2 extra nodes diametrically opposite to each other as shown in Fig. 4(b). One can see that there are \( \frac{(N+1)}{2} \) and \( \frac{(N-1)}{2} \) nodes from the original network located at the left and right half of the ring, respectively. To satisfy the demands from Node \( N + 1 \) to the nodes at the left half, we need \( \frac{(N+1)}{2} \) sub-carriers, which can also be reused for the demands from Node \( N + 1 \) to the right half. Those \( \frac{(N+1)}{2} \) sub-carriers can be reused for the traffic from Node \( N + 2 \) to the nodes at left half. Those \( \frac{(N-1)}{2} \) sub-carriers can be further reused for the traffic between Node \( N + 2 \) to the \( \frac{(N-1)}{2} \) nodes at right half, and the traffic from Node \( N + 2 \) to Node \( N + 1 \). As a result, for the new ring with \( N + 2 \) nodes, we need \( \frac{N^2-1}{8} + \frac{(N+1)}{2} + \frac{(N+2)^2-1}{8} \) sub-carriers.

Now we consider the general case with \( \mu \geq 1 \), \( GC \geq 0 \), the sub-carrier and guard-carrier allocation is equivalent to the allocation of a set of \( (X + GC) \) sub-carriers. Thus a total of \( \left\lceil (X+GC) \frac{N^2}{8} \right\rceil \) (or \( (X+GC) \frac{N^2-1}{8} \)) sub-carriers are enough in the ring network. Moreover, since the sub-carrier index is allocated incrementally, the traffic demands with maximum hop-distance \( \frac{N}{2} \) are assigned last (i.e., owning the largest sub-carrier index). The last assigned spectrum path does not need a guard-carrier. Thus \( \left\lceil (X+GC) \frac{N^2}{8} \right\rceil - GC \) (or \( (X+GC) \frac{N^2-1}{8} - GC \)) sub-carriers are sufficient, which equals the lower bound in Theorem 2.

B. \( MS \) with Predetermined Routing

1) Lower Bound: In the case that the routing path is predetermined, we can estimate the load on a given fiber \( j \) using Eq. (12), where \( J \) is the total number of spectrum paths using the fiber. Then the load \( LD \) on the most congested fiber determines the minimum number of sub-carriers on a fiber as shown in Theorem 4. This lower bound is applicable to the network with non-uniform traffic but may not be achievable due to the spectrum continuity, and sub-carrier consecutiveness constraints. In the example shown in Fig. 5, there are 4 spectrum paths along the path A-B-C-D, B-C-D-A, C-D-A-B, and D-A-B-C. With \( GC = 1 \), the load on each link is \( 3 + 2 = 5 \). All the spectrum paths overlap with the spectrum path along B-C-D-A that uses sub-carriers 3, 4. Therefore, the sub-carriers 3 and 4 cannot be used along Fiber A-B, and at least 7 sub-carriers are required. Moreover, if one extra spectrum path with 2 sub-carriers along A-B is added into the network, due to the sub-carrier consecutiveness constraint, we need at least 3 more sub-carriers with one of them as the guard-carrier.

\[
L_j = \sum_{i \in P_j} t_i + GC \times (J - 1)
\]

Theorem 4: If the routing is predetermined, and the most congested fiber has load \( LD = \max_{j \in J} L_j \), then \( MS \geq LD \).

2) Upper Bound: In the case that the paths are predetermined, we can obtain the upper bound by relaying \( MS \) with the dilation and the congestion of the network as shown in Theorem 5, where the fiber usage represents the number of various spectrum paths that use the fiber.

Theorem 5: If the routing is predetermined, the maximum fiber usage is \( R \), and the maximum path length is \( M \), then \( MS \leq ((R - 1) \times M + 1) \times (X + GC) - GC \).

Proof: Since the maximum fiber usage is \( R \), there are maximum \( R \) spectrum paths overlapping in one single fiber. We construct an interference graph [1], [14] by viewing each...
spectrum path as a vertex. Vertices are adjacent if the corresponding spectrum paths share at least one common fiber. Thus the maximum degree for the interference graph is \((R-1) + M\). According to Brook’s Theorem [15], for any connected undirected graph with maximum degree \(\Delta\), the chromatic number is at most \(\Delta + 1\). In the interference graph, we thus only requires \(\Delta + 1 = (R-1) + M + 1\) set of sub-carriers. Since each spectrum path requires at most \(X + GC\) consecutive sub-carriers, the required sub-carrier number on one fiber is bounded by \(MS \leq ((R-1) + M + 1) \times (X + GC) - GC\) after excluding the guard-carrier for the spectrum paths owns the largest index sub-carrier.

VI. Heuristic Algorithms for the RSA problem

The proposed ILP model is tractable when the problem size (e.g., network topology, traffic demands) is small. For the large scale problem, we have to rely on heuristic algorithms to obtain a practical solution within reasonable time. To achieve the goal of minimizing the maximum sub-carrier number on a fiber (i.e., \(MS\)), we propose two algorithms to choose the routing paths and maximize the reuse of sub-carriers in spectrum allocation.

A. Shortest Path with Maximum Spectrum Reuse (SPSR)

For a given set of spectrum path request pair \(SP = \{<p_i, t_i>\}\), where \(p_i\) is the path and \(t_i\) is the request size (in terms of the number of sub-carriers) of the \(i\)-th spectrum path, intuitively, the more the sub-carrier reuse can be achieved, the more we can reduce the maximum sub-carrier number. Thus we propose the shortest path with maximum spectrum reuse (SPSR) algorithm which combines the shortest path routing with the maximum reuse spectrum allocation (MRSA) algorithm shown in Algorithm 1. In Algorithm 1, the spectrum path requests are first sorted according to the size of the traffic demand. Larger traffic demand has a higher priority since the sub-carrier consecutiveness constraint makes it harder to find available consecutive sub-carriers for the larger traffic demand. Note that only fiber-disjoint spectrum paths may reuse the same sub-carriers, we hence use \(J\) to record the set of spectrum paths that are accommodated in the current iteration and employ a first-fit strategy to find available consecutive sub-carriers as shown in Line 5 and 9.

B. Balanced Load Spectrum Allocation (BLSA)

In this section, we propose another method, namely, Balanced Load Spectrum Allocation (BLSA), which determines the routing by balancing the load within the network to potentially minimize the maximum sub-carrier number on a fiber. As shown in the following 3 stages, BLSA also employs the spectrum allocation scheme in Algorithm 1.

Stage 1: Path generation. In this stage, we use the \(k\)-shortest path algorithm [17] to generate the \(k (k = 1)\) path(s), namely \(P^h_{s,d}\), where \(h = 1, 2, \ldots, k\), for each node-pair \((s, d)\).

\(^4\)In fact, this can be achieved by a greedy algorithm such as the Welsh-Powell algorithm [16].

Algorithm 1 Maximum Reuse Spectrum Allocation (MRSA)

1: Sort the spectrum path requests in the descending order of the traffic demands;
2: while There exists non-zero traffic demands do
3: \(J := \emptyset\)
4: Take the request with the maximum demands (say \(t_j\));
5: Accommodate \(t_j\) using the first available consecutive sub-carriers;
6: \(J := J \cup p_j\);
7: for all the remaining requests having non-zero traffic demands do
8: if \(p_i\) is disjoint with all the paths in \(J\) then
9: Accommodate \(<p_i, t_i>\) using the first available consecutive sub-carriers;
10: \(J := J \cup p_i\);
11: end if
12: end for
13: end while

Stage 2: Path selection. In this stage, we decide the path for each spectrum path with the goal of balancing the load among all the fibers within the network. The load of a fiber \(j (L_j)\) is estimated using Eq. (12), where \(I\) is the number of various spectrum paths using the fiber. The goodness of a path is evaluated by calculating the maximum fiber load \(LD = \max_{j \in J} L_j\) in the network. The candidate path that produces the smallest \(LD\) is used as the routing path for the corresponding spectrum path. More specifically, starting from the spectrum path with the largest traffic demand, assign one of the \(k\) paths to it while minimizing \(LD\), until all the node-pairs with non-zero traffic demands are considered.

Stage 3: Spectrum allocation. In this stage, we use Algorithm 1 to accommodate all the spectrum paths.

VII. Simulations and Performance Analysis

In this section, we present the simulation results of the proposed ILP model, heuristic algorithms and the lower bound analysis. The ILP model is implemented using the ILOG CPLEX [18].

A. ILP, Heuristic Algorithms and Lower Bound Analysis

1) ILP and Lower Bound Analysis on Ring Networks: To verify the correctness of the ILP model and the analysis, we simulate a 4-node ring \((R_4)\), and a 5-node ring \((R_5)\) network with uniform traffic demand of \(X\) sub-carriers. The maximum sub-carrier index employed among all the fibers or \(MS\) is shown in Table I. For example, in \(R_4\) with \(X = 1\) and \(GC = 2\), the CS lower bound is \(\left(1 + 2\right) \times \frac{4}{5} - 2 = 4\). We can observe that for ring networks with various uniform traffic demand and guard-carrier size, the ILP model can achieve the lower bound produced by the cut-set (CS) method. Note that in all the cases, the upper bound (UB) and the lower bound are tight as proved in Theorem 3.
on each fiber link after the spectrum allocation (i.e., after Stage 3). One can see that for most fibers, the estimated load and the required sub-carrier number are not exactly the same. This is due to the spectrum continuity constraint or the sub-carrier consecutiveness constraint as discussed in Section V. However, on the most congested fiber, the required sub-carrier number equals the estimated load, which actually shows that the BLSA algorithm achieves the lower bound in such cases. Furthermore, the comparison between BLSA and SPSR in Fig. 8 again indicates that BLSA can indeed achieve load balancing in the network.

We also compare the various combinations of traffic demands and guard-carrier size GC in Fig. 9. The x-axis is the uniform traffic demand X and y-axis is the maximum sub-carrier number among all the fibers in the network. For the same X, bigger GC implies more overhead for the guard-carrier and thus requiring more sub-carriers. Note that one may treat the guard-carrier as part of the traffic demand. Consequently, we observe that the cases with (X = 1, GC = 3), (X = 2, GC = 2), and (X = 3, GC = 1) require almost the same maximum sub-carrier number, which is indicated by the dash line in Fig. 9. The small difference among the above 3 cases is due to their difference in the guard-carrier size for the spectrum path that owns the largest sub-carrier index. In other words, the spectrum path that has the largest sub-carrier index does not need a guard-carrier above the last sub-carrier.

2) Non-uniform Traffic Demand: For the case with non-uniform traffic demands, we collect the results by randomly generating the traffic within the range [0, r], where r is the maximum traffic demands. Figure 10 shows the lower bound (LB) (using Theorem 4) for MS under the balanced load routing and shortest path routing as well as the MS by BLSA and SPSR. As one can see that the lower bound of BLSA is smaller than that of SPSR. Moreover, with SPSR, the MS is very close to the corresponding lower bound. This can be explained by the fact that the shortest path routing can potentially reduce the overall path overlapping. On the other hand, although BLSA has a smaller lower bound, the gap of MS between BLSA and its LB is larger. This is due to the longer routing paths and more overlapping from the load balance routing. In general, we can conclude that balanced routing results in smaller MS when compared to the shortest path routing. However, our other results show that the total sub-carriers consumed by BLSA is larger than that of SPSR due to the similar reasons described above.

VIII. Conclusion

The orthogonal frequency division multiplexing technology enables the possibility of a fine granularity of frequency allocation and traffic grooming in the Spectrum-sliced Elastic Optical Path (SLICE) network. In this work, we have comprehensively studied the routing and spectrum allocation (RSA) problem in the SLICE network. We have proven the NP-hardness of the optimal RSA problem, and have developed an Integer Linear Programming (ILP) model to optimally minimize the maximum sub-carrier number on a fiber in...
the SLICE network. We have also analyzed the lower/upper bounds of the maximum sub-carrier number for both general networks and ring networks. Two heuristic algorithms, namely, balanced load spectrum allocation (BLSA) and shortest path with maximum spectrum reuse (SPSR) are then proposed to efficiently solve the RSA problem in a large network. Our analysis has proven that the lower/upper bounds of the sub-carrier number on a ring network with uniform traffic is tight. Our simulations have confirmed the correctness of the static RSA studied in this work can be applicable when building a green-network. In the future, we will study the RSA problem with on-line or dynamic traffic. Topics such as how to effectively utilize the unique features of the SLICE network to implement bandwidth squeezed protection/restoration, energy efficient resource utilization in SLICE networks also deserve further study.

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