Construction Algorithms for $k$-Connected $m$-Dominating Sets in Wireless Sensor Networks

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Outline

• Introduction

• Construction algorithms
  • MDSA: 1-connected $m$-dominating
  • LDA: distributed $k$-connected $m$-dominating
  • ICGA: centralized $k$-connected $m$-dominating

• Simulation results

• Conclusion
Introduction

• Wireless Sensor Networks
• Connected Dominating Sets
• Motivation
Wireless Sensor Networks

A Wireless Sensor Network (WSN) is an ad hoc wireless network which consists of a huge amount of static or mobile sensors. The sensors collaborate to sense, collect, and process the raw information of the phenomenon in the sensing area (in-network), and transmit the processed information to the observers.
Connected Dominating Sets

A dominating set (DS) is a subset of all the nodes such that each node is either in the DS or adjacent to some node in the DS.
A connected dominating set (CDS) is a subset of the nodes such that it forms a DS and all the nodes in the DS are connected.
## Motivation

<table>
<thead>
<tr>
<th>Flooding</th>
<th>CDS</th>
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<tbody>
<tr>
<td>Redundancy</td>
<td>Reduction of communication overhead</td>
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<tr>
<td>Contention</td>
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<tr>
<td>Collision</td>
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<tr>
<td>Unreliability</td>
<td>Reliability</td>
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</table>

CDS is used as a virtual backbone in wireless networks.
Motivation

Applications of CDS: Broadcast

- Only nodes in CDS relay messages
- Reduce communication cost
- Reduce redundant traffic
Motivation

Applications of CDS: Unicast

- Only nodes in CDS maintain routing tables
- Routing information localized
- Save storage space
Motivation

CDS plays an important role in WSNs.

New challenges

Does a CDS provide fault tolerance?

Does a CDS provide routing flexibility?
Motivation

$k$-Connected $m$-Dominating Set ($kmCDS$)

- $k$-connected
  - A graph is $k$-connected if and only if it contains $k$ independent paths between any two nodes.

- $m$-dominating
  - If each node not in $C$ is dominated by at least $m$ nodes in $C$, then $C$ is an $m$-dominating set.

A $kmCDS$ provides fault tolerance and routing flexibility.
Motivation

Related works

  - 3 K-Gossip-based algorithms: $k=m$
  - CDSA: $k=2$, $m=1$, 64-approximation centralized algorithm
  - 3 centralized algorithms: $k=1$; $k=2$; $3 \leq k \leq m$
  - 1 centralized algorithm: input graph at least $\max(k, m)$-connected; not easy to implement
  - Centralized CGA: probabilistic algorithm
  - Distributed DDA: high message complexity
Construction Algorithms

- Preliminaries
- MDSA
- LDA
- ICGA
Maximal Independent Set (MIS) is a maximal set of pair-wise non-adjacent nodes.
Preliminaries

- Open neighbor set of \( v \): \( N(v) = \{ u \mid (v, u) \in E \} \)
- Closed neighbor set of \( v \): \( N[v] = N(v) \cup \{ v \} \)
- Common node set of \( u \) and \( v \): \( S_{cn}(u, v) = N[v] \cap N[u] \).
- Common black node set of \( u \) and \( v \): \( S_{cbn}(u, v) = \{ \text{All the black nodes in } S_{cn}(u, v) \} \).
- \( v \)'s local graph: \( GL(v) \) is the graph induced by \( v \) and all of its neighbors.
- Local vertex connectivity of node \( v \): \( LVC(v) = \) vertex connectivity degree of \( GL(v) \).
MDSA

General idea:

1. Construct a $(1, 1)$-CDS $C_{11} = M_1 \cup C_0$, where $M_1$ is an MIS and $C_0$ is the set of connectors connecting the nodes in $M_1$.
2. Construct a $(1, m)$-CDS $C_{1m} = C_{11} \cup M_2 \cdots \cup M_m$ where $M_i$ is an MIS from $G \setminus (C_{11} \cup M_2 \cdots \cup M_{i-1})$.

Outcome:

- The performance ratio of MDSA is $(5 + 5/m)$ for $m \leq 5$ and $7$ for $m \geq 6$.
- The time complexity of MDSA is $O(m \cdot Diam)$ and the message complexity is $O(m(\Delta + 1)|V|)$, where $\Delta$ is the maximum node degree and $Diam$ is the network diameter.
If $|P \cap Q| \geq k$ and $LVC(P) = LVC(Q) = k$, then $LVC(P \cup Q) = k$. 

**Fact**
General idea:

1. Construct a \((l, m)\)-CDS \(C_{lm}\) using MDSA.

2. Negotiate a common black node set: Every black node in \(C_{ll}\) negotiates with its parents or siblings who are also in \(C_{ll}\) about the \(S_{cbn}\) to make \(|S_{cbn}| \geq k\). (Unnecessary if \(k = 2\))

3. Build a local \(k\)-connected subgraph (Algorithm 1): Every black node in \(C_{ll}\) builds a local \(k\)-connected graph \(G_k\) which includes all the black neighbors in \(C_{lm}\) and \(S_{cbn}\), and marks all the nodes in \(G_k\) black.
LDA

Outcome:

- The performance ratio of LDA is $\max\{5/m, 1\}2\Delta$, where $\Delta$ is the maximum node degree.

- The message complexity of LDA is $O(\Delta|V|)$ and time complexity is $O((m + \Delta) \cdot Diam)$, where $\Delta$ is the maximum node degree and $Diam$ is the network diameter.
ICGA

Facts

• If $G$ is a $k$-connected graph, and $G'$ is obtained from $G$ by adding a new node $x$ with at least $k$ neighbors in $G$, then $G'$ is also a $k$-connected graph.

• Given a $k$-connected graph $G$ and a connected set $F$ which can $k$-dominate $G$, the graph $G'$ composed by $G \cup F$ is $(k + 1)$-connected.

General idea (Algorithm 2):

1. Construct a $(1,m)$-CDS $C_{1m}$ using MDSA.
2. Sequentially augment set $C_{1m}$ for $k$-connectivity to obtain a $(k,m)$-CDS.
**ICGA**

**Outcome:**

- The performance ratio of ICGA is \( f \), where

\[
 f = \begin{cases} 
 5k + \frac{5}{m} + 5H_{k-1} & m \leq 5 \\
 7k & m \geq 6 
\end{cases} \quad k \leq 6 \\
\begin{cases} 
 7k - 7 & m \leq 5 \\
 7k & m \geq 6 
\end{cases} \quad k \geq 7 
\]

and \( H_{k-1} \) is the \((k-1)\)th harmonic number.

- The time complexity of ICGA is \( O(|V|^{3.5}|E|) \).
Performance ratio of DDA


- The performance ratio of DDA is $(5 + \frac{5}{m})(k^2+1)$ for $m \leq 5$ and $7(k^2+1)$ for $m \geq 6$, if we use MDSA in phase 1 to construct a $(1,m)$-CDS $C_{1m}$. 
Simulation Results
The size of the $km$CDS obtained by LDA is 5% larger than that by DDA.

<table>
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<tr>
<th>Size of $km$CDS</th>
<th>DDA [13]</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>Message complexity</td>
<td>$O(\Delta^2</td>
<td>V</td>
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The size of the $km$CDS obtained by LDA is 25.5% and 20.5% smaller for $k=m=2$ and $k=m=3$ respectively than that by $k$-Coverage.

<table>
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<th>$k$ and $m$</th>
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Conclusion

- A $km$CDS provides fault tolerance and routing flexibility.

- **MDSA**: a distributed $(l,m)$-CDS construction algorithm

- **LDA**: a distributed $km$CDS construction algorithm for general $k$ and $m$ with low message complexity.

- **ICGA**: a centralized algorithm with a constant performance ratio and it guarantees obtaining a $km$CDS.

- Derive a tighter bound of the performance ratio of DDA [13].
Thanks

Q & A