Localized Construction of Connected Dominating Set in Wireless Networks

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## Algorithm Classification

- Distributed VS. Centralized
- Completely localized VS. Serialized

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<th>PR</th>
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<td>Wu’s O(n)</td>
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<td>Alzoubi’s</td>
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Key idea

Maximal Independent Set (MIS) is a maximal set of pair-wise non-adjacent nodes.

MIS $\leftrightarrow$ DS
Notations

Black nodes: MIS nodes.
Blue nodes: nodes connecting black nodes.
Grey nodes: dominatee nodes.
Black nodes + Blue nodes = dominating nodes (CDS)
For each node $u$

$r(u) = \text{the number of 2-hop-away neighbors} - d(u)$

where $d(u)$ is the degree of node $u$
Node $u$ with the smallest $<r, \text{deg}, \text{id}>$ within its neighborhood becomes black and broadcast a BLACK message where \text{deg} is the effective degree.
r-CDS (Cont.)

If $v$ receives a BLACK message from $u$, $v$ becomes grey and broadcasts a GREY message containing $(v, u)$. 
r-CDS (Cont.)

- black node $w$ receives a GREY message $(v, u)$
- $w$ not connected to $u$

*Color* $v$ blue
r-CDS (Cont.)

- $v$ has received a GREY message $(x, y)$
- $v$ receives a BLACK message from $u$
- $y$ & $u$ not connected

Color $v$ and $x$ blue
Analysis

Lemma 1 All the black nodes form an MIS.

- No 2 adjacent white nodes become black simultaneously.
- All the neighbors of a black node become grey.
- Black nodes form an independent set.
- Grey nodes can no longer become black.
Lemma 2 Let $d$ denote the number of hops between any pair of black nodes, then $d=2$ or $d=3$.

- Black nodes form an MIS.
- A grey node must have a black neighbor.
Analysis (Cont)

*Theorem 1* All the black nodes and blue nodes from a CDS.

*Theorem 2* The time complexity is $O(\Delta)$ where $\Delta$ is the maximum node degree and the message complexity is $O(n \Delta^2)$. 
Lemma 3: Let $S$ be any MIS of a UDG $G$. For any node $u$ in $S$, the number of the nodes in $S$ that are at most three hops away from $u$ is at most 42.

$$D = \frac{nd^2\pi}{4A(Q)}$$

$$D \leq \left[ 1 - \frac{\sqrt{3}}{2} + \sqrt{\frac{3}{4} + \frac{2\sqrt{3}}{\pi}(n - 1)} \right]^2$$

where $D$ is a maximum density of packing $n$ equal circles in another larger circle.
Analysis (Cont)

Theorem 3 The performance ratio of r-CDS is 172.

- The size of any MIS $S$ in a graph is at most $4 \cdot \text{opt} + 1$ where $\text{opt}$ is the size of any optimal CDS of the graph.

- $|C| \leq 2 \cdot 42 |S|/2 + |S| \leq 172 \cdot \text{opt} + 43$