Maximum Lifetime of Sensor Networks with Adjustable Sensing Range

A. Dhawan, C. T. Vu, A. Zelikovsky, Y. Li, S. K. Prasad

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Outline

• Background
• Problem Statement and LP formulation
• The approximation algorithm
• Greedy solution to the dual problem
• Experimental Evaluation
• Discussion
• Conclusion
Introduction

- Sensor networks
- Major constraints – energy, computation, bandwidth
Introduction

• High node density implies that only a subset of nodes need to be active.

• *Target coverage problem* – A set of targets that need to be covered.

• *Idea:* Pick a set of active sensors as a number of set covers $C_1, C_2, \ldots, C_m$ and use these one by one.

• *Question:* How long? Need to assign a *time* to each cover. Pairs $(C_m, t_m)$
Adjustable range model

• Now let's make things more interesting…
• Adjustable range – Each sensor can vary its range from 0 (off) to $MAXDIST$
• So in addition to picking the sensors $s_i$ that participate in $(C_m, t_m)$ we need to associate a range $r_i$ with each $s_i$
• Makes the problem more interesting because as range increases, target coverage increases but so does energy
Contributions

• Problem studied first by Cardei et al [4]
• We propose a different LP formulation
• Give a provably good heuristic
• Can handle non-uniform battery at each sensor
• Smooth sensing range model in place of discrete range model
• Initial results show 4 x improvement
Related work

• Cardei et al. [4]
• Maximize number of subsets – limit k

- \( c_k \), boolean variable, for \( k = 1..K \); \( c_k = 1 \) if this subset is a set cover, otherwise \( c_k = 0 \).
- \( x_{ikp} \), boolean variable, for \( i = 1..N \), \( k = 1..K \), \( p = 1..P \); \( x_{ikp} = 1 \) if sensor \( i \) with range \( r_p \) is in cover \( k \), otherwise \( x_{ikp} = 0 \).

Maximize \( c_1 + \ldots + c_K \)

subject to

\[
\sum_{k=1}^{K} (\sum_{p=1}^{P} x_{ikp} e_p) \leq E \\
\sum_{p=1}^{P} x_{ikp} \leq c_k \\
\sum_{i=1}^{N} (\sum_{p=1}^{P} x_{ikp} \cdot a_{ipj}) \geq c_k \\
x_{ikp} \in \{0, 1\} \text{ and } c_k \in \{0, 1\}
\]

\(i:\ i^{th} \) sensor, when used as index
\(j:\ j^{th} \) target, when used as index
\(p:\ p^{th} \) sensing range, when used as index
\(k:\ k^{th} \) cover, when used as index

for all \( i = 1..N \)
for all \( i = 1..N, k = 1..K \)
for all \( k = 1..K, j = 1..M \)
Sensor Network Lifetime Problem (SNLP) with range assignment

• Given a monitored region R, a set of sensors $s_1, s_2, \ldots s_m$ and a set of targets $i_1, i_2, \ldots i_n$, and energy supply $b_i$ for each sensor, find a monitoring schedule $(C_1,t_1), \ldots, (C_k,t_k)$ and a range assignment for each sensor in a set $C_i$ such that:

  (1) $t_1 + \ldots + t_k$ is maximized,

  (2) each set cover monitors all targets $i_1,\ldots,i_n$ and,

  (3) each sensor $s_i$ does not appear in the sets $C_1,\ldots,C_k$ for a time more than $b_i$.
LP formulation

Maximize: \[ \sum_{j=1}^{m} t_j \]

Subject to \[ \sum_{j=1}^{m} C_{ij} t_j \leq b_i \] \hspace{1cm} (1)

where,
- \( b_i \) is the battery for sensor \( i \),
- Rows \( i \), \( i=1, \ldots, n \) represent each sensor,
- Columns \( j \), \( j=1, \ldots, m \) represent each sensor cover,
- and, \( C_{ij} = 0 \) if sensor \( i \) is not in sensor cover \( j \),
- \( C_{ij} = g(d) \), if sensor \( i \) is in sensor cover \( j \) with a sensing range fixed to \( d \) and \( g \) is a function of energy over distance.
Example

• Suppose $m$ sensors, $p$ covers

• Maximize: $\sum tj$

Subject to:

\[
\begin{array}{cccccccc}
C_1 & C_2 & C_3 & \cdots & C_p \\
0 & f(r_2) & 0 & \cdots & b_1 \\
\end{array}
\]

\[
\leq bm
\]
Comments

• Substantially different from formulation in [4] (Max \( C_1 + C_2 + \ldots + C_k \))
• They indirectly maximize number of sets up to some limit \( k \). We directly maximize lifetime \( t \)
• Also, it can be shown that having more than \( n \) covers \( C_j \) with non-zero \( t_j \) is of no use, where \( n \) is the order of sensors
• Problem: Exponential columns in \( n \)
Garg-Könemann

• **Defn 1 – Packing LP.**

• **General form:**

\[
\max \{ c^T x \mid Ax \leq b, x \geq 0 \} \quad [8]
\]

where, \( A, b \) and \( c \) are \( (m \times n) \), \( (m \times 1) \) and \( (n \times 1) \) matrices whose entries are positive.

• **GK needs an \( f \)-approximation to finding the minimizing length column of \( A \)**

• \( \text{length}_y(j) = \sum_i A(i,j) y(i) / c(j) \) for any positive vector \( y \)
The algorithm

Input: A vector $b \in \mathbb{R}^m$, $\varepsilon > 0$, and an $f$-approximation algorithm $F$ for the problem of finding the minimum length column $A_q(j)$ of a packing LP $\{\max c^T x | Ax \leq b, x \geq 0\}$

Output: A set of columns $\{A_j^j\}_{j=1}^k$, each supplied with the value of the corresponding variable $x^j$, such that $(x^1, \ldots, x^k)$ correspond to all non-zero variables in a near-optimal feasible solution of the packing LP $\{\max c^T x | Ax \leq b, x \geq 0\}$

(1) Initialize: $\delta = (1 + \varepsilon)((1 + \varepsilon)m)^{-1/\varepsilon}$, for $i = 1, \ldots, m$, $y(i) \leftarrow \frac{\delta}{b(i)}$, $D \leftarrow m\delta$, $j = 0$

(2) While $D < 1$

   Find the column $A_q$ using the $f$-approximation $F$.
   Compute $p$, the index of the row with the minimum
   $\displaystyle \frac{b(i)}{A_q(i)}$
   $j \leftarrow j + 1$, $x^j \leftarrow \frac{b(p)}{A_q(p)}$, $A^j \leftarrow A_q$
   For $i = 1, \ldots, m$, $y(i) \leftarrow y(i) \left(1 + \varepsilon \frac{b(p)}{A_q(p) / A_q(i)}\right)$, $D \leftarrow b^T y$

(3) Output $\{(A_j^j, \frac{x^j}{\log_{1+\varepsilon} \frac{1}{\delta}})\}_{j=1}^k$
Result

THEOREM 4.1. The Lifetime problem with adjustable sensing range assignment can be approximated within a factor of \((1+\epsilon)f\), for any \(\epsilon > 0\) by using the Algorithm of Fig. 1, where \(f\) is the approximation ratio of the algorithm that picks the minimum weight column in Fig. 1.

This result is implied by the Garg-Könemann algorithm [8].

• So we need an \textit{f-approximation} to the dual problem
Minimum Weight Sensor Cover with Adjustable Sensing Range

• Given a monitored region R, a set of sensors $s_1, s_2, \ldots, s_n$ and a set of targets covered by each sensor for a range $r_i$ and the weight $w_i$ for each sensor, find the sensor cover with minimum total weight.

• So the range influences the weight

• *Basic idea:* A sensor wants the best ratio of targets covered to energy spent
A greedy algorithm

Our $f$-approximation is a greedy heuristic that tries to add sensors to the set cover by picking a sensor $s_i$ with a sensing range $r_i$ that maximizes the following ratio:

$$Gval(s_i) = \frac{\text{No. of uncovered targets covered by } s_i}{\text{weight } \times e_i}$$

Here, weight is the packing LP variable and is updated by Garg-Könemann. Also, $e_i$ is a function of the distance $d_{ij}$ between sensor $s_i$ and target $t_j$ and can be varied to study linear, quadratic and other energy models.
1. For each sensor $s_i$, compute the vector $D_i$ given below
   \[ D_i = \left[ \frac{1}{e_{i1}}, ..., \frac{m}{e_{im}} \right] \]
Here, the numerator represents the number of targets covered, and $m$ is the number of targets it can cover with range set to MAXDIST.

2. Find the maximum value of $D_i$.

3. Divide $D_i$ by $\text{weight}$.
   $\text{weight}$ is the variable from Garg-Könemann for the next step.

4. Insert $D_i/\text{weight}$ into a heap, along with $(m_i, r_p)$ which represents number of uncovered targets and the sensing range respectively.

5. Extract $P = \max(m_i^*, r_p^*)$ from the Binary Heap.

6. Update Binary Heap
   for each target $t_j$ covered by $P$
   for each sensor $i$ in the Binary Heap
      for each $\alpha \in D_i$, vector of that sensor
      if $(d_{\alpha} \geq d_{ij})$ then
         $M_{\alpha} = M_{\alpha} - 1$
         // reduce uncovered targets
      else
         break
   end for
   end for
   end for

7. Update max, rebuild Heap.

8. Repeat 2-7 until all targets are covered.
THEOREM 5.1. The Greedy Algorithm for the Minimum Weight Sensor Cover Problem with Adjustable Sensing Ranges has an approximation ratio \((1 + \ln k)\).

This is from the standard greedy algorithm for the Minimum Weight Set Cover Problem with \(k\) points to cover.

COROLLARY 5.2. The Lifetime problem with adjustable sensing range assignment can be approximated within a factor of \((1 + \varepsilon)(1 + \ln m)\) for any \(\varepsilon > 0\) by using the Algorithm of Fig. 1.

This result comes from Theorem 4.1 and Theorem 5.1 with \(k = O(m)\) elements to cover, \(m\) being the number of targets.
Experimental Results

- 100mx100m area
- Number of Sensors N: 80 to 200
- Number of targets: 25 or 50
- Range $r$: 5m to 60m
- Same as [4] but we allow range to vary smoothly instead of discrete steps
- Same energy models – linear and quadratic
- Use GK to find sensor covers. Then solve LP for assigning time to each sensor cover.
Fig 3. Variation in Network Lifetime with Number of Sensors. Number of Targets=25, Energy model is linear. AR-SC denotes the algorithm in [4].
Results

Fig 4. Variation in Network Lifetime with Number of Sensors. Number of Targets=50, Energy model is linear.
Results

Fig 5. Variation in Network Lifetime with Number of Sensors. Number of Targets=25, Energy model is quadratic.
Reasons for improvement

- Smoothly varying sensing range
- Hence, we spend energy needed to reach target and not the next step
Reasons for improvement

• Ability to assign fractional time to each cover instead of running algorithm in steps
• Provably good algorithm with approximation ration \((1+\ln m)\)
Conclusions

• New formulation
• Provably good heuristic
• Initial results indicate significant improvement
• Future work – more comparisons, distributed algorithm for the same problem