PARALLEL HEAP

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Priority Queue

DELETE & INSERT
DELETE-THINK-INSERT CYCLE

Serial-Access Heap
$O(\log n)$-TIME ACCESS

Parallel Priority Queue
# Earlier Work

<table>
<thead>
<tr>
<th>parallelization of Heap</th>
<th>#processors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quinn &amp; Yoo 1984</td>
<td>pipelined delete</td>
</tr>
<tr>
<td>Biswas &amp; Browne 1987</td>
<td>concurrent insert &amp; delete</td>
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<tr>
<td>Rao &amp; Kumar 1988</td>
<td>pipelined insert &amp; delete</td>
</tr>
<tr>
<td>Jones 1989</td>
<td>concurrent heap merging</td>
</tr>
</tbody>
</table>

## Parallelization of non-heap data structures

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Ellis 1980</td>
<td>AVL trees</td>
<td>logn</td>
</tr>
<tr>
<td>Manber 1984</td>
<td>Binary Search Tree</td>
<td>logn</td>
</tr>
</tbody>
</table>

## Hardware implementation

<p>| | | |</p>
<table>
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<th></th>
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<tbody>
<tr>
<td>Leiserson 1981</td>
<td>linear array of PE's</td>
<td></td>
</tr>
<tr>
<td>Fan &amp; Cheng 1989</td>
<td>linear array of PE's</td>
<td>(each element w-item wide)</td>
</tr>
</tbody>
</table>

*None of the above schemes can efficiently use more than logn processors*
Parallel Heap  [ICPP 1990]

NEW DATA STRUCTURE

MODEL EREW PRAM

O(p) DELETIONS IN O(log n) TIME

O(p) INSERTIONS IN O(log n) TIME

\[ n = \text{Size Of Parallel Heap} \]
\[ p = \text{Number Of Processors} \]

\[ 1 \leq p \leq n, \text{ Speedup } = O(p) \]
Serial-Access Heap

![Tree Diagram]

DELETION

Delete Update Process
(Substitute Items)

INSERTION

Insert Update Process
- Top to Bottom

(Insertion Path)

PIPELINED OPERATION
Parallel Heap

PARALLEL HEAP PROPERTY

DELETE-THINK-INSERT CYCLE \( \{ O(\log r) \} \)

PIPELINE CYCLE

\( (r/\log r = p/\log n) \)
DELETE UPDATE PROCESS

\[ r \text{ substitute items} \]

O(\log r)\text{-time process}

INSERT UPDATE PROCESS

\[ r \text{ insert items} \]

O(\log r)\text{-time process}
Parallel Heap

maintenance processors
r/\log r=2 MP's

r=4 general processors

node 1

2 MP's

79 79 80 80

2 MP's

83 87 94 95

2 MP's

95 95 97 98

2 MP's

97 97 98 99

2 MP's

83 87 91 92

2 MP's

85 86

last_node=11

insert 21 80 90 89 79 89 95 40
& delete 4 items

sort new items & merge with items at root
21 40 60 75 76 79 79 80 89 89 90 95
delete for think phase substitute items for delete proc.
insert items for insert process
delete process
sub. items 76 79 79 80

insert process 1
insert item=89 89
path=011

insert process 2
insert item=90 95
path=100

79 79 80 80

80 81 81 84

83 87 94 95
81 82 82 83
90 94 97 97
87 88 89 90

95 95 97 98
97 97 98 99
83 87 91 92
85 86

8
9
10
11
12

last_node=12

sub. item + node2 + node3
76 79 79 79 79 80 80 80 80 81 81 84

to root to node3 sub. item at node2
insert process 1
insert item = 89 89
path = 11

delete process

sub. items
80 81 81 84

83 87 94 95
81 82 82 83
90 94 97 97
87 88 89 90

76 79 79 79
79 80 80 80

95 95 97 98
97 97 98 99
83 87 91 92
85 86

8 9 10 11 12

last_node = 12
general processors

sort new items
mark last nodes for substitute items
merge new items with root
sort substitute items initiate delete and insert processes

maintenance processors

pipeline cycle 1

pipeline cycle 2

think

A delete-think-insert cycle
**Time Complexity**

**NUMBER OF PROCESSORS**

\[ p = r + (r/\log r) \times \log(n/r) \]

\[ \approx (r/\log r) \times \log n \]

\[ \log(n/r) \]

\[ r/\log r \]

\[ r/\log r \]

\[ r/\log r \]

\[ r/\log r \]

\[ r/\log r \]

\[ r/\log r \]

**O(r) operations**

**O(\log n) time**

**O((r/\log r) \times \log n) operations**

**O(\log n) time**

**O(p) operations**

**O(\log n) time**

\[ 1 \leq p \leq n, \quad \text{Speedup} = O(p) \]
Calculation of $\gamma$

$p = \gamma + \frac{\gamma}{\log r} \left\lceil \log \left( \frac{m}{n} \right) + 1 \right\rceil$

$h = 240 \quad \Rightarrow \quad \gamma = 16$

$p = 32 \quad \Rightarrow \quad \frac{\gamma}{\log r} \log n$

$32 = 16 + \frac{16}{4} \left\lceil \log (15+1) \right\rceil$

$p = 2^{12}, \quad h = 2^{16}$

$p \approx \frac{\gamma}{\log r} \log n$

$\Rightarrow \frac{\gamma}{\log r} \approx \frac{p}{\log n} = 2^8$

$\Rightarrow \gamma \approx 2850$
Problems with using maintenance processors

dedicated and partially unused

overflow and underflow
Parallel Heap without maintenance processors

\[ r = p \]

**cycle time**

- \( \log p \) levels in \( O(\log p) \) time
- \( \log(n/p) \) levels in \( \frac{\log(n/p) \times O(\log p)}{\log p} \)

**Cycle time** = \( O(\log n) \)

\( p \) inserts and \( p \) deletes in \( O(\log n) \) time

for \( 1 \leq p \leq n \)