1.1.
24. Write each of these statements in the form “if p, then q” in English. [Hint: Refer to the list of common ways to express conditional statements provided in this section.]
   d) The Red Wings will win the Stanley Cup if their goalie plays well.

   Form: \[ q \rightarrow p \]
   Given
   \[ p: \text{Red Wings’ goalie plays well} \]
   \[ q: \text{Red Wings will win the Stanley Cup} \]
   Convert to: \( p \rightarrow q \)
   If Red Wings’ goalie plays well, the Red Wings will win the Stanley Cup.

26. Write each of these propositions in the form “p if and only if q” in English.
   d) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.

   Given
   \[ p: \text{You can see the wizard} \]
   \[ q: \text{the wizard is not in} \]
   \( (p \rightarrow q) \land (q \rightarrow p) \)

27. State the converse, contrapositive, and inverse of each of these conditional statements.
   c) A positive integer is a prime only if it has no divisors other than 1 and itself.

   Given
   \[ p: \text{A positive integer is a prime} \]
   \[ q: \text{A positive integer has no divisors other than 1 and itself} \]
   converse: \( q \rightarrow p \)
   if a positive integer has no divisors other than 1 and itself, a positive integer is a prime.
   contrapositive: \( \neg q \rightarrow \neg p \)
   if a positive integer has divisors other than 1 and itself, a positive integer is not a prime.
   inverse: \( \neg p \rightarrow \neg q \)
   If positive integer is not a prime, it has divisors other than 1 and itself.

30. How many rows appear in a truth table for each of these compound propositions?
\[ d) (p \land r \land s) \lor (q \land t) \lor (r \land \neg t) \]

\[ 2^5 = 32 \]

35. Construct a truth table for each of these compound propositions.

\[ e) (p \leftrightarrow q) \lor (\neg p \leftrightarrow q) \]

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \neg p )</th>
<th>( p \leftrightarrow q )</th>
<th>( \neg p \leftrightarrow q )</th>
<th>( (p \leftrightarrow q) \lor (\neg p \leftrightarrow q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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<td>T</td>
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<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
16. An explorer is captured by a group of cannibals. There are two types of cannibals—those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.

b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.

**When I ask you, “Are you a liar will you say ‘yes’?”**

<table>
<thead>
<tr>
<th>If a:</th>
<th>Will say:</th>
</tr>
</thead>
<tbody>
<tr>
<td>knight</td>
<td>no</td>
</tr>
<tr>
<td>knave</td>
<td>yes</td>
</tr>
</tbody>
</table>

17. When three professors are seated in a restaurant, the hostess asks them: “Does everyone want coffee?”
The first professor says: “I do not know.”
The second professor then says: “I do not know.” Finally, the third professor says: “No, not everyone wants coffee.” The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

*The first professor knows that he wants coffee. The second professor knows that he wants coffee. The third professor knows that he doesn’t want coffee and the waitress understands what they want.*

28. A says “I am the knight,” B says, “A is not the knave,” and C says “B is not the knave.”

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Is it possible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>knight</td>
<td>knave</td>
<td>spy</td>
<td>A(t) ∧ B(t) ∧ C(f)</td>
</tr>
<tr>
<td>knight</td>
<td>spy</td>
<td>knave</td>
<td>A(t) ∧ B(t) ∧ C(t)</td>
</tr>
<tr>
<td>knave</td>
<td>knight</td>
<td>spy</td>
<td>A(f) ∧ B(f) ∧ C(t)</td>
</tr>
<tr>
<td>knave</td>
<td>spy</td>
<td>knight</td>
<td>A(f) ∧ B(↑t) ∧ C(t)</td>
</tr>
<tr>
<td>spy</td>
<td>knight</td>
<td>knave</td>
<td>A(↑t) ∧ B(t) ∧ C(t)</td>
</tr>
<tr>
<td>spy</td>
<td>knave</td>
<td>knight</td>
<td>A(↑t) ∧ B(t) ∧ C(f)</td>
</tr>
</tbody>
</table>
1.3.
14. Determine the truth value of each of these statements if the domain consists of all real numbers.
b) \( \exists x \ ( x^4 < x^2) \)

**True, for domain: \(-1 < x < 1\)**

d) \( \forall x \ ( 2x > x) \)

**False, only valid for domain: \(0 < x < \infty\)**

1.4.
7.c
7. Translate these statements into English, where \( C(x) \) is “\( x \) is a comedian” and \( F(x) \) is “\( x \) is funny” and the domain consists of all people.
c) \( \forall x \ ( C(x) \land F(x)) \)

**All comedians are funny.**

17. Suppose that the domain of the propositional function \( P(x) \) consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.
d) \( \forall x \ ( P(0) \land P(1) \land P(2) \land P(3) \land P(4)) \)