Scaling Laws for Cognitive Radio Network with Heterogeneous Mobile Secondary Users

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Outline

- Introduction
- System Model
- Routing and Scheduling Scheme
- Capacity and Delay Scaling Performance
- Conclusion
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Introduction-CRN

Cognitive Radio Network (CRN)

- The conflict between spectrum scarcity and the underutilization of licensed spectrum propels the study of Cognitive Radio technology.
- Cognitive Radio Network consists of the primary users licensed to the spectrum, and the secondary users which access the spectrum opportunistically.
  - The secondary network coexists with the licensed primary network.
  - The secondary users access the spectrum opportunistically without causing harmful interference to the primary users.
The secondary network coexist with the licensed primary network

- The number of primary users and secondary users
  - \( n \) primary users and \( m \) secondary users, and \( m = \Theta(n^\beta) \)
  - Whether \( \beta > 1, \beta = 1, \beta < 1 \) ?
- How “cognitive” are the secondary users?
  - How much information the secondary users know about the primary users?
  - Are the secondary users willing to relay the packets for the primary users?
- Static or mobile?
- ad-hoc or hybrid network?
Introduction-CRN

- The secondary users could access the spectrum opportunistically
  - How to limit the interference of secondary users to primary users?
    - The existence of secondary network should not degrade the performance of primary network
  - How to guarantee the transmission opportunity for the secondary users?
    - The secondary network should not suffer severe outage probability (i.e., the fraction of secondary users that cannot be served)
Introduction - Previous Works

- [S. Jeon, 2009]
  - Basic assumptions:
    - Dense network with \( n \) static primary users and \( m \) static secondary users with
      \[
      m = \Theta(n^\beta), \quad \text{and} \quad \beta > 1
      \]
    - The secondary nodes know the location of all the primary nodes
    - Define the preservation region around each primary user to limit the interference from secondary users
  - Results:
    - Primary network: \( \lambda_p(n) = \Theta\left(\frac{1}{\sqrt{n \log n}}\right) \)
    - Secondary network: \( \lambda_s = \Theta\left(\frac{1}{\sqrt{m \log m}}\right) \)

- [C. Yin, 2010]:
  - Basic assumptions:
    - The secondary users only know the location of primary transmitters
    - Define the preservation region around each active primary transmitters.
  - Results:
    - Primary network: \( \lambda_p(n) = \Theta\left(\frac{1}{\sqrt{n \log n}}\right) \quad \text{and} \quad D_p(n) = \Theta(n \lambda_p(n)) \)
    - Secondary network: \( \lambda_s(m) = \Theta\left(\frac{1}{\sqrt{m \log m}}\right) \quad \text{and} \quad D_s(m) = \Theta(m \lambda_s(m)) \)

Introduction-Previous Works

- The cooperation between PU and SU could improve the capacity and delay scaling in the CRN.
  - [L. Gao, 2009]:
    - **Basic Assumptions:**
      - The secondary users are willing to relay the primary packets.
      - There are $n$ static primary users and $m$ secondary users, either static or mobile, with $m = \Theta(n^\beta), \beta > 2$
    - **Results1:** SUs are static
      - Primary network: $\lambda_p(n) = \Theta(\frac{1}{\log n})$, and $D_p(n) = \Theta(\sqrt{n^\beta \log n \lambda_p(n)})$
      - Secondary network: $\lambda_s(m) = \Theta(\frac{1}{\sqrt{m \log m}}), D_s(m) = \Theta(m \lambda_s(m))$
    - **Results2:** SUs are mobile (i.i.d. mobility model)
      - Primary network: $\lambda_p(n) = \Theta(\frac{1}{\log n}), D_p(n) = \Theta(1)$
      - Secondary network: $\lambda_s(m) = \Theta(1), D_s(m) = \Theta(m)$

[X. Wang, 2011]:

- **Basic Assumptions:**
  - There are $n$ static primary users and $m$ mobile secondary users, with $m = \Theta(n^\beta)$, $\beta > 1$.
  - Cooperation is considered in this work.
  - Secondary users move under the hierarchical i.i.d. mobility model.

- **Results:**
  - Primary network:
  - Secondary network:

![Diagram of cognitive radio network with 1st-layer SU, 2nd-layer SU, and number of secondary users $n$]
Motivated by the fact that:

- Cooperation and Mobility could significantly improve the scaling laws in Cognitive Radio Network.
- Different mobile secondary nodes could have different moving areas in Cognitive Radio Network (Mobility Heterogeneity).
- All the previous works have some limitations in terms of system models and scaling laws.

We study:

- A more general and representative mobility model which reflects the mobility heterogeneity.
- The routing and scheduling scheme which utilize the mobility heterogeneity of secondary users.
- The impact of mobility heterogeneity on the scaling laws of Cognitive Radio Network.
Outline

- Introduction
- System Model
- Hierarchical Relay Algorithm
- Performance Of The Hierarchical Relay Algorithm
- Conclusion
Network Model:

- The primary network consists of \( n \) static, randomly and evenly distributed primary users in the unit area, which are grouped into S-D pairs one-by-one.

- The secondary network consists of \( m = (h + 1)n^{1+\varepsilon} \) heterogeneous mobile secondary users, where \( h = O(\log n), \varepsilon > 0 \) which are also grouped into S-D pairs one-by-one.

- The unit square is divided into non-overlapping small square cells, with side length \( r = \sqrt{\frac{2\log N}{N}}, N = n^{1+\varepsilon} \). Nodes can communicate with each other only when they are in the same cell.
System model – II/III

Channel Model:

- Path Loss Only:
  - Normalized channel gain is \( g(d) = d^{-r} \), where \( r > 2 \).
- We apply the Gaussian Channel Model to regulate the transmission rate, which is a continuous function to the SINR.
  - The data rate from primary transmitter \( P_i \) to primary receiver \( P_{D(i)} \):
    \[
    R(P_i, P_{D(i)}) = \log\left(1 + \frac{P_p g(\| P_i - P_{D(i)} \|)}{N_0 + I_p + I_{sp}}\right)
    \]
  - \( N_0 \): the ambient noise power
  - \( I_p \): the interference from all the other primary transmitters
  - \( I_{sp} \): the interference from all the secondary transmitters
Mobility Model for Secondary Users:

- The secondary users are uniformly and randomly distributed at the beginning.
- Each secondary user would move within a circular region centered at its initial position, according to the i.i.d. mobility model.
- The moving area of each mobile SU is $n^{-\alpha}$, where $\alpha$ follows the discrete uniform distribution:
  
  $\alpha = 0, \frac{\alpha_0}{h}, \frac{2\alpha_0}{h}, ..., \alpha_0$ with equal probability $p = \frac{1}{h + 1}$

- $h = O(\log n)$ and $\alpha_0$ is a random positive value. ($h$ and $\alpha_0$ determine the mobility heterogeneity together)

- We call the SUs with $\alpha = \frac{i\alpha_0}{h}$ the i-th type secondary user.
Mobility Model:

- Denote the k-th secondary user of type i as \( S_{i,k} \), and its initial position as \( X_{i,k} \), where \( 0 \leq i \leq h \), \( 1 \leq k \leq N = n^{1+\varepsilon} \).
- Under the proposed mobility model: \( \| S_{i,k} - X_{i,k} \| \leq R_i \), where
  \[
  R_i = \Theta(n^{-i\alpha_0/2h})
  \]
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Primary Network Routing Scheme

- The secondary users are willing to act as the relay nodes for primary users.

- The primary routing scheme would utilize the mobility heterogeneity of SUs to make the packet approach its destination progressively.

- Since the SUs with larger type would correspond to smaller moving area, thus we define the maximum type of SU that can be exploited to relay the primary packets.
  
  ➢ Definition 1: (Critical Relay Type) The critical relay type $h^*$ is defined as: $h^* = \max\{ i \mid R_i \geq 2\sqrt{2r} \}$, where $i = 0,1,2,...h$
The primary relay algorithm would utilize the SU relay nodes from type 0 to type $h^*$:

- In the first step, the primary source node would relay the packet to a 0-type SU.
- In the intermediate relay steps, the $(i-1)$-th type SU $S_{i-1,u_{i-1}}$ which holds the packet would relay it to a $i$-th type SU $S_{i,u_i}$, whose moving area contains the primary destination node. ($1 \leq i \leq h^*$)
- In the final step, the $h^*$-type SU which holds the packet would relay the packet to the primary receiver.
**Primary Relay Algorithm**

**Algorithm 1 Relay Algorithm for Primary Packet $B_p$**

**Input:** The primary source node $P_i$ and destination node $P_j$

**Output:** The $h^* + 1$ intermediate secondary relay nodes

1. $P_i$ relay $B_p$ to a 0 type SU $S_{0,u_0}$, when $S_{0,u_0}$ moves to the same cell as $P_i$.
2. for $k=0$ to $(h^* - 1)$ do
3. $S_{k,u_k}$ moves within its moving area until it meets $S_{k+1,u_{k+1}}$ in the same cell, whose initial position satisfies $\|X_{k+1,u_{k+1}} - P_j\| < R_{k+1} - \sqrt{2}r$ and $\|X_{k,u_k} - X_{k+1,u_{k+1}}\| < R_k$.
4. $S_{k,u_k}$ relay $B_p$ to $S_{k+1,u_{k+1}}$.
5. end for
6. $S_{h^*,u_{h^*}}$ moves within its moving area until it arrives at the same cell as $P_j$.
7. $S_{h^*,u_{h^*}}$ relay $B_p$ to $P_j$. 
For the secondary source node $S_{i,k_i}$ and its destination node $S_{j,k_j}$, the secondary relay algorithm would utilize SU relay nodes from type 0 to type $h' = \min\{j, h^*\}$:

- In the first step, the secondary source node would relay its packet to a 0-type SU.
- In the intermediate relay steps, the $k$-th type SU $S_{k,u_k}$ which holds the packet would relay it to a $(k+1)$-th type SU $S_{k+1,u_{k+1}}$, whose moving area contains the initial position of destination node. ($0 \leq k \leq h'-1$)
- In the final step, the $h'$-the type SU $S_{h',u_{h'}}$ which holds the packet would relay it to the destination node $S_{j,k_j}$, when they are encountered in the same cell.
### Algorithm 2: Relay Algorithm for Secondary Packet $B_{s,j}$

**Input:** The source node $S_{i,k_i}$ and destination node $S_{j,k_j}$

**Output:** The $h' + 1$ intermediate secondary relay nodes

1. $S_{i,k_i}$ moves within its moving area until it meets a 0-type SU $S_{0,u_0}$ in the same cell.
2. $S_{i,k_i}$ relay $B_{s,j}$ to $S_{0,u_0}$.
3. **for** $k=0$ to $(h' - 1)$ **do**
4. $S_{k,u_k}$ moves within its moving area until it meets $S_{k+1,u_{k+1}}$ in the same cell, whose initial position satisfies $\|X_{k+1,u_{k+1}} - X_{j,k_j}\| < R_{k+1} - \sqrt{2}r$ and $\|X_{k+1,u_{k+1}} - X_{k,u_k}\| < R_k$.
5. $S_{k,u_k}$ relay $B_{s,j}$ to $S_{k+1,u_{k+1}}$.
6. **end for**
7. $S_{h',u_{h'}}$ moves within its moving area until it encounters $S_{j,k_j}$ in the same cell.
8. $S_{h',u_{h'}}$ relay $B_{s,j}$ to $S_{j,k_j}$.
25-TDMA scheme is adopted:

- All the cells are divided into 25 sub-slots by a 5*5 pattern.
- The cells in different subsets would be activated with a round-robin fashion within one time slot.
We define preservation region to control the interference from SUs to PUs:

- The preservation region is a square that contains 9 cells, with the active primary transmitter in the center cell.
- Only SUs outside the current preservation region could transmit or relay packets.
Scheduling Scheme

- The scheduling scheme would guarantee transmission opportunity for both primary network and secondary network.

- The scheduling scheme consists of $2h^*+3$ phases, the first $h^*+1$ phases would transmit the primary packets:

  For $k = 1, 2, ..., h^*$,

  **Phase $k$:** During the active period of each cell, all pairs of nodes $(S_{k-1}, u_{k-1}, S_k, u_k)$ residing in this cell are eligible for transmission in this phase, if $S_{k-1}, u_{k-1}$ contains a primary packet $B_p$ to relay and $S_k, u_k$ can act as the $k$-th type relay SU for this $B_p$. One of such node pairs would be randomly selected to transmit if the eligible transmission node pairs is non-empty in this cell.

**Phase $(h^*+1)$:** During the active period of each cell, all pairs of nodes $(S_{h^*}, u_{h^*}, P_j)$ residing in this cell are eligible for transmission in this phase, if $S_{h^*}, u_{h^*}$ contains a primary packet $B_p$ destined to $P_j$. One of such node pairs would be randomly selected to transmit if the eligible transmission node pairs is non-empty in this cell.
Scheduling Scheme

- The next $h^* + 2$ phases would transmit secondary packets.

**Phase $(h^* + 2)$**: During the active period of each cell, randomly select a source SU $S_{i,k}$ in this cell (if there is any). Let $S_{i,k}$ relay a secondary packet $B_{s,j}$ to a random 0-type SU $S_{0,u(0)}$, if there exists any 0-type SU in this cell.

For $k = 1, 2, ..., h^*$,

**Phase $(h^* + 2 + k)$**: During the active period of each cell, two types of SU pairs residing in this cell are eligible for transmission in this phase: (1) node pair $(S_{k-1, u_{k-1}}, S_{k-1, j_{k-1}})$ which satisfies: $S_{k-1, u_{k-1}}$ contains a secondary packet $B_{s,k-1}$ destined to $S_{k-1, j_{k-1}}$; or (2) node pair $(S_{k-1, u_{k-1}}, S_{k, u_k})$ which satisfies: $S_{k-1, j_{k-1}}$ contains a secondary packet $B_{s,k'} (k \leq k' \leq h^*)$ to relay and $S_{k, u_k}$ can act as the $k$-th type relay SU for $B_{s,k'}$. One of such node pairs would be randomly selected to transmit if the eligible transmission node pairs is non-empty in this cell.

**Phase $(2h^* + 3)$**: During the active period of each cell, all pairs of nodes $(S_{h^*, u_{h^*}}, S_{j,k})$ ($h^* \leq j \leq h$) residing in this cell are eligible for transmission in this phase, if $S_{h^*, u_{h^*}}$ contains a secondary packet $B_{s,j}$ destined to $S_{j,k}$. One of such node pairs would be randomly selected to transmit if the eligible transmission node pairs is non-empty in this cell.
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Lemma 5&6: In any cell, there are at most $\Theta(1)$ PUs and $\Theta(\log n)$ SUs of each type with high probability.

Lemma 7: During the routing process of primary packets, the primary transmitters and secondary relay nodes can support constant data rate in each cell.

Theorem 1: Under the proposed relay algorithm, the primary network can achieve the following average per-node throughput with high probability: $\lambda_p = \Theta\left(\frac{1}{h}\right)$

Theorem 2: Under the proposed primary relay algorithm, the primary network can achieve the following average delay with high probability: $D_p = \Theta\left(hn^{(1+\varepsilon-h^*\frac{\alpha}{h})} + h^2n^{\frac{\alpha}{h}} \log^2 n\right)$
Lemma 9: During the routing process of secondary packets, the secondary transmitters and relay nodes can support a constant data rate in each cell.

Theorem 3: Under the proposed secondary relay algorithm, the secondary network can achieve the following per-node throughput with high probability:

\[ \lambda_s = \Theta\left(\frac{1}{h^2 \log n}\right) \]

Theorem 4: Under the proposed secondary relay algorithm, the secondary network can achieve the following average delay with high probability:

\[ D_{s,j} = O(h^2 \log n + h^2 n^h \log^2 n + h^2 n^{(1+\varepsilon^2)} \log n) \]

where \( j \) is the type of destination node.
Optimal Performance of Primary Network

- The effect of mobility heterogeneity (i.e., $h$ and $\alpha_0$) on the capacity and delay scaling laws of primary network:
  - If $h = \Theta(1)$, denote $\alpha_{th} = \frac{1 + \varepsilon - 2 \log \log n / \log n}{1 + 1/h}$, the primary network could achieve the following average delay:

$$D_p = \begin{cases} 
\Theta(n^{\frac{\alpha_0}{h}} \log^2 n), & \text{if } \alpha_0 \geq \alpha_{th} \\
\Theta(n^{1+\varepsilon-\alpha_0}), & \text{if } \alpha_0 < \alpha_{th} 
\end{cases}$$

- Despite the throughput performance is optimal, but the delay performance is still suboptimal, since $D_p = \omega(poly \log n)$
If \( h = \Theta(\log n) \), denote \( \alpha_{th'} = \frac{1 + \varepsilon - 3\log \log n / \log n}{1+1/h} \), the primary network can achieve the following average delay:

\[
D_p = \begin{cases} 
\Theta(\log^4 n), & \text{if } \alpha_0 \geq \alpha_{th'} \\
\Theta(n^{1+\varepsilon-\alpha_0} \log n), & \text{if } \alpha_0 < \alpha_{th'}
\end{cases}
\]

with the per-node throughput \( \lambda_p = \Theta\left(\frac{1}{\log n}\right) \).

In this case, the delay performance can be improved when \( \alpha_0 \) increase from 0 to \( \alpha_{th'} \), until near-optimal delay performance is achieved.
Curve for the relation between Delay/Capacity tradeoff and mobility heterogeneity:
Similar to primary network, the secondary network can also achieve the optimal performance when \( h = \Theta(\log n) \) and \( \alpha_0 \geq 1 + \varepsilon \):

- The secondary network can achieve the following average delay:

\[
D_p = \begin{cases} 
\Theta(\log^4 n), & \text{if } j \geq h^* \\
\Theta(n^{1+\varepsilon-j\alpha_0/h} \log^3 n), & \text{if } j < h^*
\end{cases}
\]

with per-node throughput of \( \lambda_s = \Theta\left(\frac{1}{\log^3 n}\right) \)

- The secondary network can achieve the near-optimal delay-capacity tradeoff for the when the type of destination SUs satisfies: \( j \geq h^* \)
Comparison with Previous Works

- Compared with [13], this paper achieves better delay scaling for secondary network while requires less secondary users.

- Compared with [14], this paper adopts more general mobility model and better scaling for secondary network.

**TABLE II: Comparison of optimal scalings of HSRM with other mobility models for SU**

<table>
<thead>
<tr>
<th>Reference</th>
<th>SU Mobility</th>
<th>PU Throughput</th>
<th>PU Delay</th>
<th>SU Throughput</th>
<th>SU Delay</th>
<th>Number of SUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. Cui [13]</td>
<td>i.i.d.</td>
<td>$\Theta(1/\log n)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(1)$</td>
<td>$\Omega(n^2)$</td>
<td>$\Omega(n^2)$</td>
</tr>
<tr>
<td>X. Wang [14]</td>
<td>hierarchical</td>
<td>$\Theta(1/\log n)$</td>
<td>$\Theta(\log^2 n)$</td>
<td>$\Theta(1/n^\delta)$</td>
<td>$\Omega(\log^2 n)$</td>
<td>$O(n^{1+\delta'})$</td>
</tr>
<tr>
<td>This paper</td>
<td>HSRM</td>
<td>$\Theta(1/\log n)$</td>
<td>$\Theta(\log^4 n)$</td>
<td>$\Theta(1/\log^3 n)$</td>
<td>$O(\log^4 n)$</td>
<td>$O(n^{1+\delta''})$</td>
</tr>
</tbody>
</table>
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Conclusion

- We propose a more general mobility model which reflects the mobility heterogeneity of secondary users in CRN.

- We show the increase of mobility heterogeneity could improve the delay-capacity tradeoff for both primary network and secondary network.

- Under optimal condition, the proposed algorithm can achieve near-constant capacity and delay scaling for primary network and part of secondary network.
Thank you!
Introduction-Previous Work

- [S. Jeon, 2009]-Preservation region

![Diagram of Preservation region with primary and secondary nodes.](image-url)

- Preservation region
- Primary node
- Secondary node