Delay Constrained Minimum Energy Broadcast in Cooperative Wireless Networks

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This slides is provided by Marjan Baghaie
1. Introduction
2. Related Work
3. System Model and Problem Formulation
4. Hardness
5. Optimal Broadcast Given Ordering
6. Approximation Algorithm
7. Performance Evaluation
None is so great that he needs no help, and none is so small that he cannot give it.

-King Solomon
Background on Wireless Networks

- A message intended for one node is heard by other nodes.
- Traditionally, in point-to-point systems, this is treated as interference.
- In cooperative systems, nodes combine information from multiple sources to decode the packet.
- **Simultaneous transmission** of the same packet by multiple sources causes collision in traditional models but could be beneficial in cooperative setting.
Motivation

- In wireless networks
  - Cooperative approaches shown to be remarkably fast*.
  - Energy is one of the precious resources.
  - Many delay sensitive applications.

- Trade-off in energy consumption and delay
  - Higher transmit power decreases the delay (faster propagation)
  - Longer delays allow lower transmission power (lower cost)

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Question

Designing efficient algorithms to identify **which** nodes should transmit, **when** and with **what** power to achieve minimum power consumption while meeting a delay constraint.

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Main Cooperative Approaches

- Coherent Signal Synchronization [1]
  - Good performance for many-to-one communication
  - Cannot handle many-to-many transmission
  - Difficult to implement

- Energy Accumulation (EA) [2,3,4]

- Mutual Information Accumulation (MIA) [5,8]
Main Cooperative Approaches

Energy Accumulation
Can decode if total received energy exceeds a threshold
\[ \sum_{s \in S(t)} p_{st} h_{sr} \geq \tau \]
- Can be implemented using Maximal Ratio Combining. [2,3]
- Significant performance improvement compared to traditional approaches. [4]
- Being implemented in industry. [6,7]

Mutual Information Accumulation
Can decode if the sum of mutual information received exceeds a threshold.
\[ \sum_{s \in S(t)} \log \left( 1 + \frac{p_{st} h_{sr}}{N} \right) \geq \mu \]
- Can be implemented using rateless codes. [9]
- More complicated to implement.
- Performs better in high SNRs. [5,8]
System Model

- Time-slotted, finite network of $n$ nodes.
- Static wireless channel, channel gain matrix $H = \{h_{ij}\}$.
- Half-duplex relay nodes, with decode and forward protocol.
- Memoryless.
- There is a delay constraint, $T$.
- Nodes can choose their transmit power dynamically.
- Unit time-slots, thus using power and energy interchangeably.
- Cooperation accumulation in the receivers: EA & MIA.
- Noise power is the same at all receivers, normalized to unity.
- Appropriate coding so that each receiving node can decode if accumulated received mutual information exceeds a given threshold, $\theta$, or $y_{rt} \geq \theta$
Delay Constrained Minimum-Energy Cooperative Transmission (DMECT)

- Transmission begins from a single source node.
- The aim is to get the message to all the nodes in a destination set $D$, with the minimum possible total energy, within a time constraint, $T$.
- Every node in the network is allowed to cooperate in the transmission, so long as they have already decoded.
- The problem becomes, which nodes should take part in cooperation, when and with what power should they transmit to achieve this aim.
Problem Formulation

\[
\begin{align*}
\min \quad & P_{\text{total}} = \sum_{t=1}^{T} \sum_{i=1}^{n} p_{it} \\
\text{s.t.} \quad & \begin{array}{l}
1. \quad p_{it} \geq 0, \quad \forall i, \forall t \\
2. \quad x_{iT+1} \geq 1, \quad \forall i \in \mathcal{D} \\
3. \quad x_{it+1} \leq \frac{1}{\theta} y_{it} + x_{it}, \quad \forall i, \forall t \\
4. \quad x_{1t} = 1, \quad \forall t \\
5. \quad x_{i1} = 0, \quad \forall i \neq 1 \\
6. \quad x_{it} \in \{0, 1\}
\end{array}
\end{align*}
\]

For EA: \( y_{it} = \log \left( 1 + \sum_{s \in S(t)} p_{st} x_{st} h_{si} \right) \)

For MIA:
\( y_{it} = \sum_{s \in S(t)} \log \left( 1 + p_{st} x_{st} h_{si} \right) \)

- \( x_{it} = 1 \) if node \( i \) is allowed to transmit, 0 otherwise.
- \( p_{it} \), transmit power
- \( y_{rt} \), information accumulated by node \( r \)
- \( T \), delay from 1 to \( n - 1 \)
### Summary of Algorithmic Results

<table>
<thead>
<tr>
<th>NEGATIVE RESULTS</th>
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<th>Mutual Information Accumulation</th>
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<tbody>
<tr>
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<td>Delay Constraint (T)</td>
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<td>Broadcast</td>
<td>$o(\log(n))$ inapprox. for $T \geq 3$</td>
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<td>$O(n)$</td>
<td>Polynomial time given ordering + ordering heuristic.</td>
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Extended version of the paper:  
http://www-scf.usc.edu/~baghaiae
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Theorem 1

The DMECB is $o(\log(n))$ inapproximable, for $T \geq 3$. 

Outline of the Proof:
Theorem 1

The DMECB is \( o(\log(n)) \) inapproximable, for \( T \geq 3 \).

Lemma 1

\[ \text{OPT}_{\text{DMECB}} \leq 1 + M \cdot \text{OPT}_{\text{SC}} + \text{OPT}_{\text{SC}} \]

Lemma 2

\[ \text{OPT}_{\text{SC}} \leq \frac{\text{OPT}_{\text{DMECB}}}{M} \]
Break the NP hard problem into three parts:

- **Ordering:**
  A node later in ordering may not transmit before the nodes earlier in ordering have decoded successfully.

- **Scheduling:**
  Deciding at which time slots the nodes should transmit.

- **Power allocation:**
  Deciding the transmit power of each node.

Given ordering, we can have a polynomial time algorithm to simultaneously solve the scheduling and power allocation problems.
**Instantaneous Power Allocation:**

\[ \text{CP}(\{1...k\}, \{k + 1...j\}, \theta, H) : \]

\[
\begin{align*}
\min \quad & \sum_{s \in S(t)} p_{st} \\
\text{s.t.} \quad & p_{st} \geq 0, \quad \forall s \\
& y_{rt} \geq \theta, \quad \forall r \in R(t)
\end{align*}
\]

where

- For EA: \( y_{it} = \log \left(1 + \sum_{s \in S(t)} p_{st} h_{si}\right) \)
- For MIA: \( y_{it} = \sum_{s \in S(t)} \log \left(1 + p_{st} h_{si}\right) \)

This is a convex program for MIA and a linear program for EA.
The solution is a deterministic dynamic program based on the following recursion:

\[
C(j, t) = \min_{k \in (1, \ldots, j)} [C(k, t - 1) + CP(\{1\ldots k\}, \{k + 1\ldots j\}, \theta, H)]
\]

where
- \(C(j, t)\) minimum energy needed to cover up to node \(j\) in \(t\) steps or less.
- \(C(k, 1) = CP(1, \{2\ldots k\}, \theta, H)\)
- \(C(1, t) = 0 \ \forall t\).

Makes at most \(O(n^2 T)\) calls to the CP solver, each of which takes polynomial time.

Thus, total minimum cost for covering \(n\) nodes by time \(T\) can be found in polynomial time by calculating \(C(n, T)\).
Theorem 4

In DMECU, with EA, there exists a solution consisting of a simple path between source and destination, which is optimum.

- Intuition behind the proof:

Corollary

The optimal solution is the weight of the shortest path given by Dijkstra’s algorithm, in the case where there is no delay.
Optimal Unicast with Energy Accumulation

Optimal DMEU with EA:

\[ C(i, t) = \min_{k \in N_r(i)} [C(k, t-1) + w(k \rightarrow i)] \]

where

- \( C(i, t) \) be the min cost for \( s \) to turn on \( i \), possibly using relays, within at most \( t \) time slots.
- \( w(k \rightarrow i) \) power it takes for \( k \) to turn on \( i \) using direct transmission.
- \( C(s, t) = 0 \), for all \( t \)
- \( C(i, 1) = w(s \rightarrow i) \)

This algorithm has a running time of \( O(n^2 T) \).

Optimal DMEU (with EA) can be found by calculating \( C(d, T) \), in polynomial time.
Approximation Results

- Bounded diameter directed steiner tree approximation algorithm* offers a factor of $O(n^c)$.
- These algorithms can be applied to DMECB through an appropriate reduction.

**Theorem 5**

For any fixed $T > 0$, there is an algorithm which runs in time $n^{O(T)}$ and gives an $O(T \log^2(n))$ approximation of the DMECB with EA.

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Performance Evaluation

Network of n nodes, uniform on a 15 by 15 square surface.

Static wireless channel, independent exponentially distributed channel gains.

Dijkstra’s used for ordering.

Optimal calculated using exhaustive search.

Dijkstra provides good heuristic for ordering.

\[ f_{h_{ij}}(h_{ij}) = \frac{1}{h_{ij}} \exp \left( \frac{h_{ij}(k)}{h_{ij}} \right) \]

\[ h_{ij} = d_{ij}^{-\eta} \]
Cooperative vs Non-Cooperative

- Smart non-cooperative algorithm used for comparison.
- Non-coop needs to solve a weighted set cover at each step, thus $o(\log(n))$ inapproximable even given ordering.
- Use greedy algorithm to give $O(\log(n))$ approx.
- Significant improvement observed in cooperative case.

Power-delay Trade-off

- Power-delay trade-off for various network densities and varying channel conditions.
- Highlights the sensitivity of dense networks and those with poor channel conditions to delay constraint.
- Showing the need for employing smart algorithms to minimize energy consumption.
Summary

Conclusion
- Formulated the problem of delay-constrained minimum energy transmission in cooperative networks.
- Studied the computational complexity of the problem.
- Provided polynomial time solution for special cases.
- Investigated the energy-delay tradeoffs inherent in cooperative networking.

Ongoing and future work
- Performing more comprehensive simulations.
- Finding tighter approximation bounds.
- Developing decentralized algorithms.
- Considering multiple simultaneous flows.

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