Exercises

1. What is the probability that a card selected at random from a standard deck of 52 cards is an ace?
2. What is the probability that a fair die comes up six when it is rolled?
3. What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?
4. What is the probability that a randomly selected day of the leap year (with 366 possible days) is in April?
5. What is the probability that the sum of the numbers on two dice is even when they are rolled?
6. What is the probability that a card selected at random from a standard deck of 52 cards is an ace or a heart?
7. What is the probability that when a coin is flipped six times in a row, it lands heads up every time?
8. What is the probability that a five-card poker hand contains the ace of hearts?
9. What is the probability that a five-card poker hand does not contain the queen of hearts?
10. What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?
11. What is the probability that a five-card poker hand contains the two of diamonds, the three of spades, the six of hearts, the ten of clubs, and the king of hearts?
12. What is the probability that a five-card poker hand contains exactly one ace?
13. What is the probability that a five-card poker hand contains at least one ace?
14. What is the probability that a five-card poker hand contains cards of five different kinds?
15. What is the probability that a five-card poker hand contains two pairs (that is, two of each of two different kinds and a fifth card of a third kind)?
16. What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?
17. What is the probability that a five-card poker hand contains a straight, that is, five cards that have consecutive kinds? (Note that an ace can be considered either the lowest card of an A-2-3-4-5 straight or the highest card of a 10-J-Q-K-A straight.)
18. What is the probability that a five-card poker hand contains a straight flush, that is, five cards of the same suit of consecutive kinds?
*19. What is the probability that a five-card poker hand contains cards of five different kinds and does not contain a flush or a straight?
20. What is the probability that a five-card poker hand contains a royal flush, that is, the 10, jack, queen, king, and ace of one suit?
21. What is the probability that a fair die never comes up an even number when it is rolled six times?
22. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 37?
23. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 77?
24. Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding 100.
   a) 30.
   b) 36.
   c) 42.
   d) 48.
25. Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding 100.
   a) 50.
   b) 52.
   c) 56.
   d) 60.
26. Find the probability of selecting none of the correct six integers in a lottery, where the order in which these integers are selected does not matter, from the positive integers not exceeding 100.
   a) 40.
   b) 48.
   c) 56.
   d) 64.
27. Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding 100.
   a) 40.
   b) 48.
   c) 56.
   d) 64.
28. In a superlottery, a player selects 7 numbers out of the first 80 positive integers. What is the probability that a person wins the grand prize by picking 7 numbers that are among the 11 numbers selected at random by a computer?
29. In a superlottery, players win a fortune if they choose the eight numbers selected by a computer from the positive integers not exceeding 100. What is the probability that a player wins this superlottery?
30. What is the probability that a player of a lottery wins the prize offered for correctly choosing five (but not six) numbers out of six integers chosen at random from the integers between 1 and 40, inclusive?
31. Suppose that 100 people enter a contest and that different winners are selected at random for first, second, and third prizes. What is the probability that Kumar, Janice, and Pedro each win a prize if each has entered the contest?
32. What is the probability that Michelle wins one of the prizes if she is one of the contestants?
33. Suppose that 100 people enter a contest and that different winners are selected at random for first, second, and third prizes. What is the probability that Kumar, Janice, and Pedro each win a prize if each has entered the contest?
34. What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 50 people enter a contest and
   a) no one can win more than one prize.
   b) winning more than one prize is allowed.
35. What is the probability that Bo, Colleen, Jeff, and Rohini win the first, second, third, and fourth prizes, respectively, in a drawing if 50 people enter a contest and
   a) no one can win more than one prize.
   b) winning more than one prize is allowed.
35. In roulette, a wheel with 38 numbers is spun. Of these, 18 are red, and 18 are black. The other two numbers, which are neither black nor red, are 0 and 00. The probability that when the wheel is spun it lands on any particular number is $\frac{1}{38}$.
   a) What is the probability that the wheel lands on a red number?
   b) What is the probability that the wheel lands on a black number twice in a row?
   c) What is the probability that the wheel lands on 0 or 00?
   d) What is the probability that in five spins the wheel never lands on 0 or 00?
   e) What is the probability that the wheel lands on one of the first six integers on one spin, but does not land on any of them on the next spin?

36. Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?

37. Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

38. Two events $E_1$ and $E_2$ are called **independent** if $p(E_1 \cap E_2) = p(E_1)p(E_2)$. For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.
   a) $E_1$: tails comes up with the coin is tossed the first time; $E_2$: heads comes up when the coin is tossed the second time.
   b) $E_1$: the first coin comes up tails; $E_2$: two, and not three, heads come up in a row.
   c) $E_1$: the second coin comes up tails; $E_2$: two, and not three, heads come up in a row.
   (We will study independence of events in more depth in Section 7.2.)

39. Explain what is wrong with the statement that in the Monty Hall Three-Door Puzzle the probability that the prize is behind the first door you select and the probability that the prize is behind the other of the two doors that Monty does not open are both $\frac{1}{2}$, because there are two doors left.

40. Suppose that instead of three doors, there are four doors in the Monty Hall puzzle. What is the probability that you win by not changing once the host, who knows what is behind each door, opens a losing door and gives you the chance to change doors? What is the probability that you win by changing the door you select to one of the two remaining doors among the three that you did not select?

41. This problem was posed by the Chevalier de Méré and was solved by Blaise Pascal and Pierre de Fermat.
   a) Find the probability of rolling at least one six when a fair die is rolled four times.
   b) Find the probability that a double six comes up at least once when a pair of dice is rolled 24 times. Answer the query the Chevalier de Méré made to Pascal asking whether this probability was greater than $\frac{1}{2}$.
   c) Is it more likely that a six comes up at least once when a fair die is rolled four times or that a double six comes up at least once when a pair of dice is rolled 24 times?

### 7.2 Probability Theory

#### Introduction

In Section 7.1 we introduced the notion of the probability of an event. (Recall that an event is a subset of the possible outcomes of an experiment.) We defined the probability of an event $E$ as Laplace did, that is,

$$p(E) = \frac{|E|}{|S|}$$

the number of outcomes in $E$ divided by the total number of outcomes. This definition assumes that all outcomes are equally likely. However, many experiments have outcomes that are not equally likely. For instance, a coin may be biased so that it comes up heads twice as often as tails. Similarly, the likelihood that the input of a linear search is a particular element in a list, or is not in the list, depends on how the input is generated. How can we model the likelihood of events in such situations? In this section we will show how to define probabilities of outcomes to study probabilities of experiments where outcomes may not be equally likely.

Suppose that a fair coin is flipped four times, and the first time it comes up heads. Given this information, what is the probability that heads comes up three times? To answer this and