Rules of Inference

Section 1.6
Section Summary

- Valid Arguments
- Inference Rules for Propositional Logic
- Using Rules of Inference to Build Arguments
- Rules of Inference for Quantified Statements
- Building Arguments for Quantified Statements
Revisiting the Socrates Example

We have the two premises:
- “if you have a current password, then you can log onto the network”
- “you have a current password”

And the conclusion:
- “you can log onto the network”

How do we get the conclusion from the premises?
The Argument

- **Argument**: a sequence of statements that end with a conclusion

- **Former example:**
  - **Premises**: “if you have a current password, then you can log onto the network” and “you have a current password”
  - **Conclusion**: “you can log onto the network”
    - p: “You have a current password”
    - q: “You can log onto the network.”

\[
\begin{array}{c}
p \rightarrow q \\
p   \\
\hline
\therefore q
\end{array}
\]

\[
((p \rightarrow q) \land p) \rightarrow q
\]
Valid Arguments

- **Valid arguments**: the conclusion or final statement of the argument must follow from the truth of the preceding statements or premises of the argument.
  - the argument is true if and only if it is impossible for all the premises to be true and the conclusion to be false.

- The rules of inference are the essential building block in the construction of valid arguments.
  - Arguments in Propositional Logic
  - Arguments in Predicate Logic
Arguments in Propositional Logic

Definition 1: An argument in propositional logic is a sequence of propositions. All but the final proposition are called premises. The last statement is the conclusion. The argument is valid if the premises imply the conclusion.

An argument form in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is valid no matter what propositions are substituted into its propositional variables in its premises, the conclusion is true if the premises are all true.

- If the premises are $p_1, p_2, ..., p_n$ and the conclusion is $q$ then ($p_1 \land p_2 \land ... \land p_n \rightarrow q$) is a tautology.
Rules of Inference for Propositional Logic

- *rules of inference*: the validity of some relatively simple argument forms
- rules of inference can be used as **building blocks** to construct more complicated valid argument forms
Modus ponens

Example:
- p : “It is snowing.”
- q : “I will study discrete math.”

Premises:
- “If it is snowing, then I will study discrete math.”
- “It is snowing.”

Conclusion:
- “Therefore, I will study discrete math.”

Corresponding Tautology:
\((p \land (p \rightarrow q)) \rightarrow q\)
Modus Tollens

Example:
- p be “it is snowing.”
- q be “I will study discrete math.”

Premises:
- “If it is snowing, then I will study discrete math.”
- “I will not study discrete math.”

Conclusion:
- “Therefore, it is not snowing.”

Corresponding Tautology:
\[
(\neg q \land (p \rightarrow q)) \rightarrow \neg p
\]
Hypothetical Syllogism

Example:
- Let $p$ be “it snows.”
- Let $q$ be “I will study discrete math.”
- Let $r$ be “I will get an A.”

Premises:
- “If it snows, then I will study discrete math.”
- “If I study discrete math, I will get an A.”

Conclusion:
- “Therefore, If it snows, I will get an A.”

Corresponding Tautology:
\[ ((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r) \]
Disjunctive Syllogism

Premises:
- \( p \lor q \)
- \( \neg p \)

Conclusion:
- \( \therefore q \)

Example:
- Let \( p \) be “I will study discrete math.”
- Let \( q \) be “I will study English literature.”

Premises:
- “I will study discrete math or I will study English literature.”
- “I will not study discrete math.”

Conclusion:
- “Therefore, I will study English literature.”

Corresponding Tautology:
\[
(\neg p \land (p \lor q)) \rightarrow q
\]
Addition

<table>
<thead>
<tr>
<th>p</th>
<th>Corresponding Tautology: p → (p ∨ q)</th>
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<tbody>
<tr>
<td>∴ p ∨ q</td>
<td></td>
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</table>

Example:
- Let p be “I will study discrete math.”
- Let q be “I will visit Las Vegas.”

Premises:
- “I will study discrete math.”

Conclusion:
- “Therefore, I will study discrete math or I will visit Las Vegas.”
**Simplification**

- **Example:**
  - Let $p$ be “I will study discrete math.”
  - Let $q$ be “I will study English literature.”

- **Premises:**
  - “I will study discrete math and English literature”

- **Conclusion:**
  - “Therefore, I will study discrete math.”

**Corresponding Tautology:**

$$(p \land q) \to p$$
Conjunction

Example:
- Let $p$ be “I will study discrete math.”
- Let $q$ be “I will study English literature.”

Premises:
- “I will study discrete math.”
- “I will study English literature.”

Conclusion:
- “Therefore, I will study discrete math and I will study English literature.”

Corresponding Tautology:
$((p) \land (q)) \rightarrow (p \land q)$
Resolution

Example:
- Let \( p \) be “I will study discrete math.”
- Let \( r \) be “I will study English literature.”
- Let \( q \) be “I will study databases.”

Premises:
- “I will not study discrete math or I will study English literature.”
- “I will study discrete math or I will study databases.”

Conclusion:
- “Therefore, I will study databases or I will study English literature.”

Corresponding Tautology:
\[
((\neg p \lor r) \land (p \lor q)) \rightarrow (q \lor r)
\]
## Rules of inference

<table>
<thead>
<tr>
<th>Rule of Inference</th>
<th>Tautology</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p ) ( \rightarrow q ) ( \therefore q )</td>
<td>( (p \land (p \rightarrow q)) \rightarrow q )</td>
<td>Modus ponens</td>
</tr>
<tr>
<td>( \neg q ) ( p \rightarrow q ) ( \therefore \neg p )</td>
<td>( (\neg q \land (p \rightarrow q)) \rightarrow \neg p )</td>
<td>Modus tollens</td>
</tr>
<tr>
<td>( p \rightarrow q ) ( q \rightarrow r ) ( \therefore p \rightarrow r )</td>
<td>( ((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r) )</td>
<td>Hypothetical syllogism</td>
</tr>
<tr>
<td>( p \lor q ) ( \neg p ) ( \therefore q )</td>
<td>( ((p \lor q) \land \neg p) \rightarrow q )</td>
<td>Disjunctive syllogism</td>
</tr>
<tr>
<td>( p \lor q ) ( \therefore p \rightarrow (p \lor q) )</td>
<td></td>
<td>Addition</td>
</tr>
<tr>
<td>( p \land q ) ( \therefore p )</td>
<td>( (p \land q) \rightarrow p )</td>
<td>Simplification</td>
</tr>
<tr>
<td>( p \land q ) ( \therefore p \land q )</td>
<td>( (p \land q) \rightarrow (p \land q) )</td>
<td>Conjunction</td>
</tr>
<tr>
<td>( p \lor q ) ( q ) ( \therefore p \lor q )</td>
<td>( ((p \lor q) \land \neg p \lor r) \rightarrow (q \lor r) )</td>
<td>Resolution</td>
</tr>
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</table>
Valid Arguments

- Example 1: From the single proposition
  \[ p \land (p \rightarrow q) \]
  Show that q is a conclusion.

- Solution:

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
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</thead>
<tbody>
<tr>
<td>1. ( p \land (p \rightarrow q) )</td>
<td>Premise</td>
</tr>
<tr>
<td>2. ( p )</td>
<td>Conjunction using (1)</td>
</tr>
<tr>
<td>3. ( p \rightarrow q )</td>
<td>Conjunction using (1)</td>
</tr>
<tr>
<td>4. ( q )</td>
<td>Modus Ponens using (2) and (3)</td>
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</tbody>
</table>
Valid Arguments

Example 2: With these hypotheses:

“It is not sunny this afternoon and it is colder than yesterday.”
“We will go swimming only if it is sunny.”
“If we do not go swimming, then we will take a canoe trip.”
“If we take a canoe trip, then we will be home by sunset.”

Using the inference rules, construct a valid argument for the conclusion:

“We will be home by sunset.”

Solution:

1. Choose propositional variables:
   
   - p : “It is sunny this afternoon,”  q : “It is colder than yesterday,”  r : “We will go swimming,”  s : “We will take a canoe trip,” and t : “We will be home by sunset.”

2. Translation into propositional logic:
   
   - Premises: \( \neg p \land q, r \rightarrow p, \neg r \rightarrow s, \) and \( s \rightarrow t \)
   
   - Conclusion: \( t \)
Valid Arguments

- **p**: “It is sunny this afternoon,”  
- **q**: “It is colder than yesterday,”  
- **r**: “We will go swimming,”  
- **s**: “We will take a canoe trip,” and  
- **t**: “We will be home by sunset.”

- **Premises:**  \( \neg p \land q, r \rightarrow p, \neg r \rightarrow s, \text{ and } s \rightarrow t \)
- **Conclusion:** t

3. **Construct the Valid Argument**

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
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<tbody>
<tr>
<td>1.</td>
<td>( \neg p \land q )</td>
</tr>
<tr>
<td>2.</td>
<td>( \neg p )</td>
</tr>
<tr>
<td>3.</td>
<td>( r \rightarrow p )</td>
</tr>
<tr>
<td>4.</td>
<td>( \neg r )</td>
</tr>
<tr>
<td>5.</td>
<td>( \neg r \rightarrow s )</td>
</tr>
<tr>
<td>6.</td>
<td>( s )</td>
</tr>
<tr>
<td>7.</td>
<td>( s \rightarrow t )</td>
</tr>
<tr>
<td>8.</td>
<td>( t )</td>
</tr>
</tbody>
</table>
Rules of Inference for predicate Logic

- Universal Instantiation (UI)

\[
\forall x P(x) \\
\therefore P(c)
\]

- Example:
  - Our domain consists of all dogs and Fido is a dog.

Premises:
  - “All dogs are cuddly.”

Conclusion:
  - “Therefore, Fido is cuddly.”
Rules of Inference for predicate Logic

- **Universal Generalization (UG)**
  
  \[
  P(c) \text{ for an arbitrary } c \\
  \therefore \forall x P(x)
  \]

- **Existential Instantiation (EI)**
  
  \[
  \exists x P(x) \\
  \therefore P(c) \text{ for some element } c
  \]

- **Existential Generalization (EG)**
  
  \[
  P(c) \text{ for some element } c \\
  \therefore \exists x P(x)
  \]
Using Rules of Inference

Example 1: Using the rules of inference, construct a valid argument to show that “John Smith has two legs” is a consequence of the premises: “Every man has two legs.” and “John Smith is a man.”

Solution:
- Let $M(x)$ denote “$x$ is a man” and $L(x)$ “$x$ has two legs” and let John Smith be a member of the domain.
- Valid Argument:

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\forall x(M(x) \rightarrow L(x))$</td>
<td>Premise</td>
</tr>
<tr>
<td>2. $M(J) \rightarrow L(J)$</td>
<td>UI from (1)</td>
</tr>
<tr>
<td>3. $M(J)$</td>
<td>Premise</td>
</tr>
<tr>
<td>4. $L(J)$</td>
<td>Modus Ponens using (2) and (3)</td>
</tr>
</tbody>
</table>
Using Rules of Inference

Example 2: Use the rules of inference to construct a valid argument showing that the conclusion

“Someone who passed the first exam has not read the book.”

goes from the premises

“A student in this class has not read the book.”

“Everyone in this class passed the first exam.”

Solution:

- Let C(x) denote “x is in this class,” B(x) denote “x has read the book,” and P(x) denote “x passed the first exam.”
- First we translate the premises and conclusion into symbolic form.

\[
\begin{align*}
\exists x (C(x) \land \neg B(x)) \\
\forall x (C(x) \rightarrow P(x)) \\
\therefore \exists x (P(x) \land \neg B(x))
\end{align*}
\]

Continued on next slide →
Using Rules of Inference

Valid Argument:

<table>
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<tbody>
<tr>
<td>1. $\exists x (C(x) \land \neg B(x))$</td>
<td>Premise</td>
</tr>
<tr>
<td>2. $C(a) \land \neg B(a)$</td>
<td>EI from (1)</td>
</tr>
<tr>
<td>3. $C(a)$</td>
<td>Simplification from (2)</td>
</tr>
<tr>
<td>4. $\forall x (C(x) \rightarrow P(x))$</td>
<td>Premise</td>
</tr>
<tr>
<td>5. $C(a) \rightarrow P(a)$</td>
<td>UI from (4)</td>
</tr>
<tr>
<td>6. $P(a)$</td>
<td>MP from (3) and (5)</td>
</tr>
<tr>
<td>7. $\neg B(a)$</td>
<td>Simplification from (2)</td>
</tr>
<tr>
<td>8. $P(a) \land \neg B(a)$</td>
<td>Conj from (6) and (7)</td>
</tr>
<tr>
<td>9. $\exists x (P(x) \land \neg B(x))$</td>
<td>EG from (8)</td>
</tr>
</tbody>
</table>
Combining rules of inference for propositions and quantified statements

- **Universal modus ponens (MP):** combines universal instantiation and modus ponens into one rule

\[ \forall x (P(x) \rightarrow Q(x)) \]

\[ P(a), \text{ where } a \text{ is a particular element in the domain} \]

\[ \therefore Q(a) \]
Returning to the Socrates Example

\[ \forall x (\text{Man}(x) \rightarrow \text{Mortal}(x)) \]

\[ \text{Man} (\text{Socrates}) \]

\[ \therefore \text{Mortal} (\text{Socrates}) \]

Step | Reason
--- | ---
1. \( \forall x (\text{Man}(x) \rightarrow \text{Mortal}(x)) \) | Premise
2. \( \text{Man}(\text{Socrates}) \rightarrow \text{Mortal}(\text{Socrates}) \) | UI from (4)
3. \( \text{Man}(\text{Socrates}) \) | Premise
4. \( \text{Mortal}(\text{Socrates}) \) | MP from (2) and (3)
Introduction to Proofs

Section 1.7
Section Summary

- Mathematical Proofs
- Forms of Theorems
- Direct Proofs
- Indirect Proofs
  - Proof of the Contrapositive
  - Proof by Contradiction
Proofs of Mathematical Statements

- A proof is a valid argument that establishes the truth of a statement.

- Proofs have many practical applications:
  - verification that computer programs are correct
  - establishing that operating systems are secure
  - enabling programs to make inferences in artificial intelligence
  - showing that system specifications are consistent
Some Terminology

- A **theorem** is a statement that can be shown to be true using:
  - definitions
  - other theorems
  - axioms (statements which are given as true)
  - rules of inference

- A **lemma** is a ‘helping theorem’ or a result which is needed to prove a theorem.

- A **corollary** is a result which follows directly from a theorem.

- Less important theorems are sometimes called **propositions**.

- A **conjecture** is a statement that is being proposed to be true. Once a proof of a conjecture is found, it becomes a theorem. It may turn out to be false.
Many theorems assert that a property holds for all elements in a domain, such as the integers, the real numbers, or some of the discrete structures that we will study in this class. Often the universal quantifier (needed for a precise statement of a theorem) is omitted by standard mathematical convention.

For example, the statement:

“If $x > y$, where $x$ and $y$ are positive real numbers, then $x^2 > y^2$”

really means

“For all positive real numbers $x$ and $y$, if $x > y$, then $x^2 > y^2$.”
Methods of proving theorems

- Direct proofs
- Proof by contraposition
- Proofs by contradiction
Even and Odd Integers

- Definition: The integer $n$ is **even** if there exists an integer $k$ such that $n = 2k$, and $n$ is **odd** if there exists an integer $k$, such that $n = 2k + 1$. Note that every integer is either even or odd and no integer is both even and odd.
Proving Conditional Statements: $p \rightarrow q$

- **Direct Proof:** Assume that $p$ is true. Use rules of inference, axioms, and logical equivalences to show that $q$ must also be true.

- **Example:** Give a direct proof of the theorem “If $n$ is an odd integer, then $n^2$ is odd.”

- **Solution:**
  - Assume that $n$ is odd. Then $n = 2k + 1$ for an integer $k$. Squaring both sides of the equation, we get:
    - $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2r + 1$,
    - where $r = 2k^2 + 2k$, an integer.
  - We have proved that if $n$ is an odd integer, then $n^2$ is an odd integer.

( 🔴 marks the end of the proof. Sometimes QED is used instead.)
Proving Conditional Statements: \( p \rightarrow q \)

**Proof by Contraposition:** Assume \( \neg q \) and show \( \neg p \) is true also. This is sometimes called an **indirect proof** method. If we give a direct proof of \( \neg q \rightarrow \neg p \) then we have a proof of \( p \rightarrow q \).

**Example:** Prove that if \( n \) is an integer and \( 3n + 2 \) is odd, then \( n \) is odd.

**Solution:**

**Contraposition:** if \( n \) is even, then \( 3n+2 \) is even

- Assume \( n \) is even. So, \( n = 2k \) for some integer \( k \). Thus \( 3n + 2 = 3(2k) + 2 = 6k + 2 = 2(3k + 1) = 2j \) for \( j = 3k + 1 \)

- Therefore \( 3n + 2 \) is even. Since we have shown \( \neg q \rightarrow \neg p \), \( p \rightarrow q \) must hold as well. If \( n \) is an integer and \( 3n + 2 \) is odd (not even), then \( n \) is odd (not even).
Proving Conditional Statements: $p \rightarrow q$

- **Proof by Contradiction**: Because the statement $r \land \neg r$ is a contradiction whenever $r$ is a proposition, we can prove that $p$ is true if we can show that $\neg p \rightarrow (r \land \neg r)$ is true for some proposition $r$.

- **Example**: Prove that if you pick 22 days from the calendar, at least 4 must fall on the same day of the week.

- **Solution**: Assume that no more than 3 of the 22 days fall on the same day of the week. Because there are 7 days of the week, we could only have picked 21 days. This contradicts the assumption that we have picked 22 days.
Theorems that are Biconditional Statements

- To prove a theorem that is a biconditional statement, that is, a statement of the form $p \iff q$, we show that $p \rightarrow q$ and $q \rightarrow p$ are both true.

- Example: Prove the theorem: “If $n$ is an integer, then $n$ is odd if and only if $n^2$ is odd.”

- Solution:
  - We have already shown (previous slides) that both $p \rightarrow q$ and $q \rightarrow p$. Therefore we can conclude $p \iff q$.
  - Sometimes iff is used as an abbreviation for “if an only if,” as in “If $n$ is an integer, then $n$ is odd iff $n^2$ is odd.”
Looking Ahead

- If direct methods of proof do not work:
  - We may need a clever use of a proof by contraposition.
  - Or a proof by contradiction.
  - In the next section, we will see strategies that can be used when straightforward approaches do not work.
  - In Chapter 5, we will see mathematical induction and related techniques.
  - In Chapter 6, we will see combinatorial proofs
Section Summary

- Proof by Cases
- Existence Proofs
  - Constructive
  - Nonconstructive
- Disproof by Counterexample
- Nonexistence Proofs
- Uniqueness Proofs
- Proof Strategies
- Proving Universally Quantified Assertions
- Open Problems
Proof by Cases

- To prove a conditional statement of the form:
  \[(p_1 \lor p_2 \lor \ldots \lor p_n) \rightarrow q\]

- Use the tautology
  \[\left[(p_1 \lor p_2 \lor \cdots \lor p_n) \rightarrow q\right] \iff \left[(p_1 \rightarrow q) \land (p_2 \rightarrow q) \land \cdots \land (p_n \rightarrow q)\right]\]

- Each of the implications \[p_i \rightarrow q\] is a case.
Proof by Cases

Example: Let $a @ b = \max\{a, b\} = a$ if $a \geq b$, otherwise $a @ b = \max\{a, b\} = b$. Show that for all real numbers $a, b, c$ $(a @ b) @ c = a @ (b @ c)$ (This means the operation @ is associative.)

Proof: Let $a$, $b$, and $c$ be arbitrary real numbers. Then one of the following 6 cases must hold.

1. $a \geq b \geq c$
2. $a \geq c \geq b$
3. $b \geq a \geq c$
4. $b \geq c \geq a$
5. $c \geq a \geq b$
6. $c \geq b \geq a$

Case 1: $a \geq b \geq c$
- $(a @ b) = a$, $a @ c = a$, $b @ c = b$
- Hence $(a @ b) @ c = a = a @ (b @ c)$
- Therefore the equality holds for the first case.

Case 2:

... 

A complete proof requires that the equality be shown to hold for all 6 cases. But the proofs of the remaining cases are similar.
Example: Show that if $x$ and $y$ are integers and both $x \cdot y$ and $x + y$ are even, then both $x$ and $y$ are even.

Suppose $x$ and $y$ are not both even. Then, one or both are odd. **Without loss of generality**, assume that $x$ is odd. Then $x = 2m + 1$ for some integer $k$.

Case 1: $y$ is even. Then $y = 2n$ for some integer $n$, so $x + y = (2m + 1) + 2n = 2(m + n) + 1$ is odd.

Case 2: $y$ is odd. Then $y = 2n + 1$ for some integer $n$, so $x \cdot y = (2m + 1)(2n + 1) = 2(2m \cdot n + m + n) + 1$ is odd.

We only cover the case where $x$ is odd because the case where $y$ is odd is similar. The use phrase **without loss of generality** (WLOG) indicates this.
Existence Proofs

- Proof of theorems of the form $\exists x P(x)$

- Constructive existence proof:
  - Find an explicit value of $c$, for which $P(c)$ is true.
  - Then $\exists x P(x)$ is true by Existential Generalization (EG).

- Example: Show that there is a positive integer that can be written as the sum of cubes of positive integers in two different ways:

  Proof: $1729$ is such a number since

  $1729 = 10^3 + 9^3 = 12^3 + 1^3$

- Nonconstructive existence proof: assume no $c$ exists which makes $P(c)$ true and derive a contradiction.
Counterexamples

- An element for which $P(x)$ is false is called a counterexample of $\forall x \ P(x)$

- Example: “Every positive integer is the sum of the squares of 3 integers.” The integer 7 is a counterexample. So the claim is false.
Uniqueness Proofs

- Theorems assert the existence of a unique element with a particular property, $\exists! x \ P(x)$. The two parts of a uniqueness proof are
  - Existence: We show that an element $x$ with the property exists.
  - Uniqueness: We show that if $y \neq x$, then $y$ does not have the property.

**Example:** Show that if $a$ and $b$ are real numbers and $a \neq 0$, then there is a unique real number $r$ such that $ar + b = 0$.

**Solution:**
- Existence: The real number $r = -b/a$ is a solution of $ar + b = 0$ because $a(-b/a) + b = -b + b = 0$.
- Uniqueness: Suppose that $s$ is a real number such that $as + b = 0$. Then $ar + b = as + b$, where $r = -b/a$. Subtracting $b$ from both sides and dividing by $a$ shows that $r = s$. 
To prove theorems of the form $\forall x P(x)$, assume $x$ is an arbitrary member of the domain and show that $P(x)$ must be true. Using UG it follows that $\forall x P(x)$.
Universally Quantified Assertions

- Example: An integer $x$ is even if and only if $x^2$ is even.
- Solution: The quantified assertion is $\forall x \ [x \text{ is even } \iff x^2 \text{ is even}]

We assume $x$ is arbitrary.

- Case 1. If $x$ is even then $x^2$ is even using a direct proof.
  - If $x$ is even then $x = 2k$ for some integer $k$.
  - Hence $x^2 = 4k^2 = 2(2k^2)$ which is even since it is an integer divisible by 2.
  - This completes the proof of case 1.

- Case 2. If $x^2$ is even then $x$ must be even (the if part or sufficiency).
  We use a proof by contraposition. Assume $x$ is not even and then show that $x^2$ is not even.
  - If $x$ is not even then it must be odd. So, $x = 2k + 1$ for some $k$. Then $x^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ which is odd and hence not even. This completes the proof of case 2.

- Since $x$ was arbitrary, the result follows by UG. Therefore we have shown that $x$ is even if and only if $x^2$ is even.
Example 1: Can we tile the standard checkerboard using dominos?
Solution: Yes! One example provides a constructive existence proof.
**Tilings**

Example 2: Can we tile a checkerboard obtained by removing one of the four corner squares of a standard checkerboard?

Solution:
- Our checkerboard has $64 - 1 = 63$ squares.
- Since each domino has two squares, a board with a tiling must have an even number of squares.
- The number 63 is not even.
- We have a contradiction.
Additional Proof Methods

Later we will see many other proof methods:

- **Mathematical induction**, which is a useful method for proving statements of the form $\forall n \ P(n)$, where the domain consists of all positive integers.
- **Structural induction**, which can be used to prove such results about recursively defined sets.
- **Combinatorial proofs** use counting arguments.