Waveband Converters Based on Four-Wave Mixing in SOAs

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Abstract—Four wave mixing (FWM) is distinguished from other wavelength conversion techniques by its ability to simultaneously convert a number of input wavelength channels. In this case, optical signal-to-noise ratio (OSNR) is insufficient to describe the performance of the device as many effects are involved. A multiwavelength FWM model is used here to simulate a waveband converter (WBC). The numerical model predicts the waveform of the FWM product. Based on that output, the Q factor of the signal and the power penalty induced to the signal can be calculated to evaluate the performance of such a device. Meanwhile, an analytical model is used for the calculation of the signal power levels and the standard deviation of the fluctuation; hence, it describes the constituent effects—namely, the extinction ratio (ER) degradation, the OSNR degradation, the gain modulation (GM) related crosstalk, and interference. The model’s validity is tested against the numerical results. To the best of the authors’ knowledge, this is the first time that a numerical model and an analytical model are used to systematically investigate a WBC and to identify the specific effects and derive the design rules. These rules are tested in the experiment. Finally, a tunable WBC (TWBC) based on the dual-pump configuration is described and implemented experimentally.

Index Terms—Four-wave mixing optical wavelength conversion, semiconductor optical amplifiers, waveband converters.

I. INTRODUCTION

The simultaneous conversion of many wavelengths is a very distinct feature of wave mixing techniques and four-wave mixing (FWM) in particular [1]–[5]. This attribute indicates that a single waveband converter (WBC) can replace a number of wavelength converters (WC), reducing the component count in an optical communication system [1]. At the same time, more flexible wavelength and waveband routing systems can be constructed if WBCs are available [6].

Multiwavelength (waveband) FWM has been proposed for a WBC [7], [8], and as a midspan inversion technique [9]–[12]. Experimental demonstrations of a single-pump waveband conversion based on FWM in optical fibers have been previously reported [7], [9]. Although a wide tunability may not be an issue when it is used for a midspan inversion, the long lengths of the nonlinear fiber required for efficient FWM makes this technique unattractive for practical implementa-

A four-channel conversion using a single pump configuration in a semiconductor optical amplifier (SOA) has been demonstrated in [12] as an optical phase conjugator. A polarization insensitive FWM operation in SOA has been presented in [13] and [14]. In [15], we have suggested a two-channel widely tunable waveband conversion (TWBC) in a dual pump configuration.

Apart from the FWM, other wave mixing techniques have been proposed in the literature for waveband conversion. Among these techniques, the differential frequency generation (DFG) offers numerous unique advantages like wide tunability [16] and format-transparent conversion [17]–[19] without spontaneous emission and operation with a pump out of the band of the involved signals [1]. The technique is free of satellite signals; however, low loss waveguides are difficult to fabricate. The high scattering losses lead to very low conversion efficiencies for the DFG and related techniques.

The WBCs based on FWM in SOAs bear some special features compared with the other schemes. A number of mechanisms have been suggested to account for the FWM [4], and they are all related to the gain and the refractive index dynamics in SOAs. These include the carrier population pulsations (CPP), the carrier heating (CH), and the spectral hole burning (SHB); the relative strength of which is significant for the features of the FWM products [5]. The spectral arrangement of the pump and the channels at the output of a four-channel WBC is shown schematically in Fig. 1. The figure shows the spectrum consisting of the input pump P1 at a frequency \( \omega_p \) and four signal channels S1, S2, S3, and S4 at frequencies \( \omega_1, \omega_2, \omega_3, \) and \( \omega_4 \), respectively. The corresponding output FWM products are shown in the same picture (F1, F2, F3, and F4).

There are a number of impairments that degrade the quality of a signal when converted through an FWM WBC. Some of

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Fig. 1. Single-pump, four-channel WBC output spectrum.
them are related to the FWM process, and they are equivalent to the impairments induced by an FWM WC [20], while the rest are attributed to the multiwavelength nature of this function. These are summarized as follows:

1) limited efficiency and low OSNR of the output signals.
   This is poorer compared to the single wavelength FWM case due to the saturation of the SOA by all signals;

2) data-induced nonlinear interference that induces pattern effects [20];

3) cross gain modulation (GM) of the signals into the SOA.
   The gain of the SOA is modulated by all the signals simultaneously. This is copied on both the pump and the signals, affecting all the FWM products as a GM noise;

4) interference with secondary FWM products that fall into the band of the product under investigation or even in the band of the initial signals and the pump;

5) imperfect filtering of the pump;

6) interference between the backward reflected signals and the forward propagating ones. These effects are not considered here, as it is shown in [21] that they are negligible;

7) extinction ratio (ER) degradation due to the saturation of the SOA gain;

8) wavelength dependent efficiency introduces a variation of the performance, reflected mainly on the OSNR between the different channels in a band.

Evidently, there is a plethora of effects that have to be accounted for when the performance of such a converter is evaluated. The OSNR estimation alone is insufficient for the evaluation of a WBC, as there are other noise sources that degrade the signal-to-noise ratio. To assess all these impairments, the incurred power penalty has to be calculated. In this paper, a numerical and an analytical model are implemented for the investigation of the WBC based on the FWM in the SOAs. This paper is arranged as follows. The numerical model is explained in detail in Section II. The product waveform is used to evaluate the Q factor of the signal and, from that, the ER-related penalty, the OSNR-related penalty, and the noise-related penalty could be calculated and compared to the overall penalty. Furthermore, the analytical formulas for the evaluation of the penalty effects are described in Section III and the experimental setup in Section IV. The analytical model can give the output signal powers and fluctuations and based on those, the ER-related penalty, the OSNR-related penalty, the interference-related penalty, and the GM-related penalty can be calculated. With some assumptions, the overall penalty can be calculated. An investigation of the two-channel WBC is performed in Section V. The TWBC is explained and investigated in Section VI. Finally, the general design rules are described and discussed in Section VII, and conclusions are given in Section VIII.

II. NUMERICAL MODEL FOR THE THEORETICAL INVESTIGATION OF WBCs BASED ON FWM IN SOAS

The theoretical model for waveband conversion based on FWM in SOAs is developed according to the methods discussed in [20]–[23]. We assume an SOA cavity with a number of electric fields propagating along z. If only the fundamental transverse electric mode propagates in the cavity; the propagation characteristics can be obtained by solving the wave equation as follows:

$$\frac{\partial}{\partial z} A_j + \frac{1}{v_g} \frac{\partial}{\partial z} A_j = \frac{i}{\omega} \frac{\partial}{\partial z} \left[ \chi^{(1)} A_j + \frac{1}{\varepsilon_o} P_{NLj} \right]$$

(1)

where $n$ is the modal refractive index, $\varepsilon_o$ is the vacuum permitivity, $c$ is the velocity of light in vacuum, and $t$ is the time. The SOA is considered as an isotropic device, so the total induced polarization of the medium is a function of the susceptibility $\chi^{(1)}(N, \omega)$ and the nonlinear polarization $P_{NL}$. $N$ is the carrier density of the semiconductor, and $\omega$ is the angular frequency of the wave. The index $j$ counts the fields that coexist in the cavity, $\omega_j = 2\pi f_j$ is their angular frequency, $\beta_j = n\omega_j/c$ is their propagation constant, and $A_j$ is the complex-varying amplitude of each field, whose envelope remains spatially invariant in the $x, y$ plane, normalized in such a way that $A_j A_j^*$ has units of power. $v_g$ is the group velocity, and its dispersion can be neglected as no walk-off effects between pulses are examined here. The linear susceptibility is related to the material gain as follows [20], [23]:

$$\chi^{(1)}(N, \omega) = -\frac{nc}{\omega_j} (i g^m_{N,j} + \alpha \Gamma g(N, \omega_j))$$

(2)

where $g^m$ is the modal gain for the $j$th mode, $\alpha$ is the linewidth enhancement factor, $\Gamma$ is the confinement factor, and $g$ is the material gain. The nonlinear polarization is induced by all possible beatings between the electric fields in the cavity. In order to describe this interaction, the most common approach uses the density matrix formalism, which is only necessary when examining a fast phenomenon like pulse interaction within an amplifier [4]. Here, the durations of the signal pulses are longer than the intraband timescales (see Table I). Hence, the phenomenological approach can be followed [20] but extended for all possible beatings between fields like in [23]. The nonlinear polarization is then given by

$$\frac{1}{\varepsilon_o} \sum_j P_{NLj}$$

$$= \frac{nc}{2\omega_j} \left( i g^m_{N,i} + \alpha \Gamma g(N, \omega) \right)$$

$$\times \left[ \frac{1}{P_{CPP}} \sum_l \sum_k h_{CPP}(\omega_k - \omega_j) E_l(t) E_l^*(t) \right] E$$

$$- \frac{(i + \alpha_{shb})}{2} g^m_{N,i}$$

$$\times \varepsilon_{shb} \sum_l \sum_k h_{SHB}(\omega_k - \omega_j) E_l(t) E_l^*(t) \right] E$$

$$- \frac{(i + \alpha_{sh})}{2} g^m_{N,i}$$

$$\times \sum_l \sum_k h_{CPP}(\omega_k - \omega_j) E_l(t) E_l^*(t) \right] E$$

(3)
where \((\omega_k - \omega_j)\) is the frequency separation between any two interacting electric fields, and the summation is over the whole set of electric fields. CPP, CH, and SHB have been introduced through the response functions of the gain, the carrier temperature, and the distribution of the intraband population, as in [21]. These functions are denoted by \(h_{\text{CPP}}, h_{\text{CH}}, \) and \(h_{\text{SHB}}, \) respectively. \(\varepsilon_{\text{CPP}} = 1/P_{\text{CPP}}, \varepsilon_{\text{CH}}, \) and \(\varepsilon_{\text{SHB}}\) are the strengths of the nonlinear processes, which are equal to the inverse saturation powers of each process [4]. \(\alpha_{\text{CH}}\) and \(\alpha_{\text{SHB}}\) are the equivalents of the linewidth enhancement factors for CH and SHB. The frequency response of the individual nonlinear processes is given by [4]

\[
\begin{align*}
    h_{\text{CPP}}(\omega_k - \omega_j) &= \frac{1}{(1 + (\omega_k - \omega_j)\tau_{\text{CPP}})(1 + (\omega_k - \omega_j)\varepsilon_{\text{CPP}})} \\
    h_{\text{CH}}(\omega_k - \omega_j) &= \frac{1}{(1 + (\omega_k - \omega_j)\tau_{\text{CH}})(1 + (\omega_k - \omega_j)\varepsilon_{\text{CH}})} \\
    h_{\text{SHB}}(\omega_k - \omega_j) &= \frac{1}{(1 + (\omega_k - \omega_j)\tau_{\text{SHB}})} \tag{4}
\end{align*}
\]

where \(\tau_x, \) with \(x = \{\text{CPP, CH and SHB}\}, \) is the characteristic timescale associated with each process. Specifically, for the CPP process [23]: \(1/\tau_{\text{CPP}} = 1/\tau_s + P_o\Gamma g_o/\varepsilon_\omega A_x \) and \(P_{\text{CPP}} = \varepsilon_\omega A_x/(\tau_{\text{CPP}}\Gamma g_o), \) where \(\varepsilon_\omega\) is the energy that corresponds to \(\omega, \) \(P_o\) is the total input power, \(g_o\) is the gain parameter, \(A_x\) is the cross-sectional area, and \(\tau_s\) is the carrier lifetime. \(1/P_{\text{CPP}}\) is the nonlinear strength of the CPP associated with the saturation power of the amplifier \(P_{\text{sat}} = \varepsilon_\omega A_x/(\tau_s\Gamma g_o). \) The relaxation times and strengths for CH and SHB are shown in Table I. Equation (3) is now simplified and explicitly contains the beating between the different electric fields. When replacing (4) in (3), it can be broken down to a set of differential equations, the general form of which are described in (5), shown at the bottom of the next page, where \(\Delta \beta\) is the summation of the propagation constants and is assumed that \(\Delta \beta = 0 \) [4]. Note that this is the general form that describes the evolution of the electric field, together with the evolution of FWM products. No assumptions regarding the relative powers between the pumps and the signals apply here [4]. The frequencies of all products can be calculated from the condition: \(\sum_l \sum_m \sum_r \sum_j \omega_l + \omega_m - \omega_r - \omega_j = 0.\)

The first term of (5) is related to the gain and the phase difference experienced by a signal at frequency \(\omega_j.\) The other terms can be divided into two cases: when \(\omega_m = \omega_r,\) which is the static effect (gain compression) and when \(\omega_m \neq \omega_r,\) which is the dynamic effect (beating related). In contrast to these two cases, the dynamic effects are related to all nonlinear mechanisms. Again, they can be separated into \(\omega_l = \omega_j,\) where the effects are related to the interference between the generated waves and the injected ones and the mixing effects where \(\omega_l \neq \omega_j\) that are related to a conventional FWM.

The material gain of the SOA is described by the parabolic model [22]

\[
g(N, \lambda_o) = a_o(N - N_o) - a_1(\lambda_o - \lambda_N)^2 \\
\lambda_N = \lambda_o - a_2(N - N_o) \tag{6}
\]

where \(N\) is the carrier density, \(N_o\) is the carriers at transparency, and \(\lambda_o\) is the wavelength that corresponds to the frequency. The model parameters are listed in Table I. The modal gain then is calculated by [22]

\[
g_{\text{N,j}}^m = \Gamma (g(N, \lambda_o) - a_o) - (1 - \Gamma)a_c - a_{\text{scat}} \tag{7}
\]

where \(a_o\) is the loss in the active layer, \(a_c\) is the loss in the claddings, and \(a_{\text{scat}}\) are the scattering losses. For the solution of (9), the SOA is divided into many sections; hence, the longitudinal variation of gain, the refractive index, and the amplified spontaneous emission (ASE) are all accounted for in this model. The propagation equations are coupled by the rate
equations for carrier density \( N \). In each section the rate equation \( q \) is solved as follows:

\[
\frac{dn_q}{dt} = \frac{I}{eV} - (AN_q + BN_q^2 + CN_q^3) - \sum_k \frac{\Gamma g(N_q, \lambda_0)}{h\omega_k} I_k - \frac{\Gamma g_m,p}{n_g} \beta_{\text{eff}} S_{\text{av,sp}}^q
\]

where \( N_q \) is the carrier density of the section \( q \), \( I \) is the injection current, \( e \) is the electron charge, and \( V \) is the active region volume [25]. The second term yields the carrier lifetime as \( R(N) = N/\tau_s \). \( I_j \) is the average intensity in section \( q \) given by

\[
(I_j(z,t)) = \frac{|A_j|^2}{A_\text{e}} + \frac{|A_j|^2}{2A_\text{e}}
\]

The model takes into account the ASE by the means of an effective spontaneous emission [25]. In the last term of (12), \( g_{m,p} \) is the peak gain, \( \beta_{\text{eff}} \) is the effective spontaneous emission factor, and \( S_{\text{av,sp}} \) is the average spontaneous photon density generated in a given section [20]. The propagation of the ASE into the SOA is investigated separately [20].

A. Model Implementation

To calculate the penalties incurred in a WBC, the waveform of the output product should be calculated, and for that, the numerical model is deployed with two input signals, S1 and S2, and one pump P1 and \( \Delta \omega = \Delta \omega_1 = 1.5 \text{ nm} \) (see Fig. 1). The accuracy of the model has been checked elsewhere [20]. The two signals are modulated with long data sequences [27 pseudo-random binary sequence (PRBS) - 128 b]. For bit rates beyond 10 Gb/s, even longer pattern sequences must be deployed to ensure that the carrier lifetime effects are accounted for. In

\[
\frac{\partial A_j}{\partial z} + \frac{1}{v_g} \frac{\partial A_j}{\partial t} = \left( \frac{ig_m}{2} - i\alpha \Gamma g(N_{j,i}) \right) (A_j) + \ldots
\]

\[
\times \left[ -\frac{1}{P_{\text{cpp}}} \left( \sum_l \left( \frac{g_{N_1}^m - i\alpha \Gamma g(N_{1,i})}{2} \right) \sum_r \sum_m h_{\text{cpp}}(\omega_m - \omega_r)A_m(t)A_r^*(t)(A_1) \right) \right] + \ldots
\]

\[
- \varepsilon_{\text{shb}} \left( \sum_l \left( \frac{1 - i\alpha_{\text{shb}}}{2} g_{N_1}^m \right) \sum_r \sum_m h_{\text{shb}}(\omega_m - \omega_r)A_m(t)A_r^*(t)(A_1) \right) + \ldots
\]

\[
- \varepsilon_{\text{ch}} \left( \sum_l \left( \frac{1 - i\alpha_{\text{ch}}}{2} g_{N_1}^m \right) \sum_r \sum_m h_{\text{ch}}(\omega_m - \omega_r)A_m(t)A_r^*(t)(A_1) \right) e^{(i\Delta \beta_{1+m-r-j} z)}
\]

Fig. 2(a), the data patterns of S1 and S2 are shown for a specific time window of 200 ps with arbitrary normalization. (b) Corresponding FWM waveforms F1 and F2 are illustrated for the same time window where crosstalk and cross modulation effects are evident. (c) Output P1, S1, and S2 with evident crosstalk effects and arbitrary normalization.

Fig. 2. (a) Data patterns of input S1 and S2 are shown for a specific time window of 200 ps with arbitrary normalization. (b) Corresponding FWM waveforms F1 and F2 are illustrated for the same time window where crosstalk and cross modulation effects are evident. (c) Output P1, S1, and S2 with evident crosstalk effects and arbitrary normalization.
In order to recognize the dynamic effects, a “sampled” pattern is plotted. A configuration with ∆ω = 2∆ω1 has been simulated, and in this case, the interference effects are minimized. The “sampled” patterns of F1, S1, and S2 have been plotted for every bit in Fig. 3(a). The x-axis corresponds to the bit number. The crosstalk effects are mainly due to the modulation of the gain. In Fig. 3(b), a histogram based on the sampled pattern of Fig. 3(a) is presented. Evidently, the level of modulation of the gain can be calculated for every bit in Fig. 3(a). The Nth level of “one” PF1 consists of two sublevels, a feature that is typical of the existence of either the GM effects [26] or interferers [27]. Assuming that both levels have a Gaussian distribution, one can calculate the standard deviations σ1 and σ0. The Gaussian approach overestimates the distribution around the power levels of “one” and “zero” [26], [27].

The same procedure is followed to plot the simulation results from the model when S1 and S2 are modulated at 160-Gb/s [see Fig. 3(c)]. The pattern effects and the cross GM effects are now “smoother” compared to the 10-Gb/s case. Both are related to the carrier lifetime, which is longer than the bit duration. In Fig. 3(d), the histogram based on the sampled pattern of the amplitudes is plotted together with the equivalent distribution. The distribution is now more Gaussian like, and the overall deviation is smaller due to the smoother pattern effects at this bit rate.

The numerical model can simulate the signal waveforms at the output of the WBC based on an FWM in the SOAs. To assess the performance of the conversion technique, one has to compare the input signal with the FWM product, and this can be done by means of the Q factor; hence, a power penalty [24]. From the numerically calculated FWM product waveforms like the ones in the figures above, the following are deduced: the average ⟨PF1⟩ and ⟨PF1⟩0, which are the level of “one” and “zero” that can be sued for the average power and output ER, calculated as ERout = ⟨PF1⟩/⟨PF1⟩0. Furthermore, the standard deviation of these levels is associated with the relative intensity noise (RIN) [24]. However, the sources of the crosstalk and, hence, the RIN cannot be identified (e.g., either a GM or an interference related crosstalk), as well as the average ASE levels in order to calculate the OSNR. Based on the above and on the formulas of [24], which are presented in Section III, the power penalties at a bit error rate of 10−9 can be calculated. To understand the involvement of the different effects in the degradation of the signal, the power penalties of the constituent effects are calculated separately. It is assumed that the main sources of degradation are 1) ER degradation; 2) spontaneous emission noise (OSNR degradation); and 3) noise due to other combined effects like interference, which are the GMs that manifest themselves as fluctuations. The third case includes all the crosstalk effects that distort the signal waveform and can be treated as an RIN. Corresponding power penalty calculations, namely, the ER-related penalty, the OSNR-related penalty, and the crosstalk-related penalty can then be performed and compared to the overall penalty. The latter can only be calculated numerically, as shown in [24].

III. ANALYTICAL MODEL

The analytical model presented in Appendix [20], [29] is used to describe an FWM-based converter of ζ signals and ξ pumps. In contrast with the time-consuming numerical calculations of the previous section, here, the model is used to provide formulas for the power levels of PF1 and PF10, the ASE power, and the level fluctuations. We assume the case where ζ = 2 and ξ = 1, the overall input power comprises the power of input signals (PS1 and PS2) and pumps (PP1). The gain saturation and the ASE noise are now enhanced. The output power of PF1 is given by (A.2)

\[
PF1 = PP1^2PS1G^3R(2ω_{p1} - ω_{s1})
\]

where according to (A.3),

\[
G = b1(PP1 + PS1 + PS2)^{-7}.
\]

Although the input power range of the model is small, it can be used to evaluate the penalties that are incurred by different effects.

If the ζ channels have the same average power Pave and the same PS1 and PS10, the calculation of PF1 and PF10 (or any of the PFk, with k = 1, ..., ζ, the index for the input signal and i = 1, 0 the index for identifying the bit) relies on averaging the gain that affects PF1 and the PF10

\[
\langle PF1 \rangle = \langle PP1^2PS1G^3R(2ω_{p1} - ω_{s1}) \rangle
\]

or

\[
\langle PF1 \rangle = \langle PP1^2PS1(G_1^3R(2ω_{p1} - ω_{s1})) \rangle
\]

similarly, the calculations for \(PF10 = PP1^2PS0(G_0^3R(2ω_{p1} - ω_{s1}))\). In order to calculate the average \(G^3\) when the signal is “one,” say \(G_1^3\), one has to take into consideration the power carried by the rest of the channels that simultaneously modulate the gain. The power of any of the signals Sk can either take PSk or PSk0. There are 2ζ−1 different combinations for the rest ζ − 1 input channels, i.e., there are 2ζ−1 possible sets of (ζ − 1) comprising PSk1 or PSk0 values. Now if there are j channels out of these ζ − 1 input channels that bear PSk1 and the rest of the channels bear PSk0, then the \((ζ − 1)!/(j!(ζ − 1 − j)!)\) out of these 2ζ−1 possible cases will give the same aggregate input
power. It is the aggregate power that is involved in the calculation of \( \langle G^3 \rangle \) [see (A.3)]

\[
\langle G^3 \rangle = \frac{1}{2^{(\zeta-1)}} \sum_{j=0}^{\zeta-1} \left( \begin{array}{c} (\zeta-1)! \\ j!(\zeta-1-j)! \end{array} \right) G_{i,j}^3
\]  

(11)

where \( G_{i,j} = a \langle PP1 + jPS1 \rangle + \langle (\zeta - 1 - j)PS1_0 + PS1 \rangle \langle \zeta - 1 - j \rangle \) -0.7, because PS1_0 = PS1_1 and PSk_0 = PS1_0. Thus, by combining (10) with (11), the average power of level "1," \( \langle PF1 \rangle \) and level “0” \( \langle PF1_0 \rangle \) is calculated.

The scope of this section is to analytically establish the calculated power penalty formulas in order to investigate the performance of the WBC. Here, the above formulas are going to be used in well-defined penalty calculation formulas in order to develop an analytical model that can efficiently describe the FWM converters.

**ER Penalty:** This penalty is related to the fact that the average values of PF1 and PF1_0 will be dependent on the average gain. Hence, the ER_{out} for the F1 depends on the power, i.e., the symbols (i = 1 or 0) of the rest of the \( \zeta - 1 \) channels

\[
\langle PF1 \rangle = PP1_2 \cdot PS1 \langle G^3 \rangle R \quad \text{and} \quad \langle PF1_0 \rangle = PP1_2 \cdot PS1_0 \langle G^3 \rangle R.
\]  

(12)

The \( ER_{out} = \langle PF1 \rangle / \langle PF1_0 \rangle \) will then be given by

\[
ER_{out} = \frac{PP1_2 \cdot PS1 \cdot 1/2^{(\zeta-1)} \sum_{j=0}^{\zeta-1} \left( \begin{array}{c} (\zeta-1)! \\ j!(\zeta-1-j)! \end{array} \right) G_{1,j}^3 R}{PP1_2 \cdot PS1_0 \cdot a \cdot 1/2^{(\zeta-1)} \sum_{j=0}^{\zeta-1} \left( \begin{array}{c} (\zeta-1)! \\ j!(\zeta-1-j)! \end{array} \right) G_{0,j}^3 R} = \frac{ER \sum_{r=0}^{\zeta-1} \left( \begin{array}{c} \frac{r}{\zeta} + r + \frac{(\zeta-1-r)}{ER} + 1 \\ \frac{r}{\zeta} + \frac{(\zeta-1-r)}{ER} + 1 \end{array} \right)^{-2.4}}{\sum_{j=0}^{\zeta-1} \left( \begin{array}{c} \frac{r}{\zeta} + \frac{(\zeta-1-r)}{ER} + 1 \\ \frac{r}{\zeta} + \frac{(\zeta-1-r)}{ER} + 1 \end{array} \right)^{-2.4}}
\]  

(13)

where \( a \) is the ratio PS1_1/PP1, and ER is the input ER. The worst \( ER_{out} \) value stems when all the \( \zeta \) channels are modulated with the same pattern. The overall power when \( i = 1 \) will be \( \zeta \cdot PS1_1 \) and when \( i = 0 \), becomes \( \zeta \cdot PS1_0 \). Thus, the output ER will be

\[
ER_{out}^{min} = ER \left( \frac{\zeta}{\zeta + \frac{\zeta}{ER}} \right)^{-2.4}
\]  

(14)

Equations (13) or (14) are the first analytical formulas to describe the ER of a signal at the output of the WBC. Then, it can be used to calculate the induced penalty (ER-related penalty) in [24]

\[
\delta_{ER} = 10 \log_{10} \left( \frac{1 + ER_{out}}{ER - 1} \right) \left( \frac{ER - 1}{1 + ER} (ER_{out} - 1) \right).
\]  

(15)

**Interference Penalty:** This penalty is related to the interferometric crosstalk between the product under investigation (say F1) that is created due to the mixing of P1 and S1 and some product that might be created between two signals S1 and S2 and falls in the band of F1 (when, e.g., \( \Delta \omega = \Delta \omega 1/2 \)). As a result of this parameter, the Q factor is now reduced in the presence of the intensity noise and in order to maintain the same Q factor, the received power must be increase. To calculate the interference penalty arising from these cases, the analysis in [26] is followed and \( r_{RIN} \) has to be calculated where \( r_{RIN} \) is the RIN. One secondary product arising from the mixing of two signals, defined as \( PF^2_k \), is assumed and interferes with F1.

\[
r^2_{int} = \sum_{k=1}^{2PF^2_k \cdot PF1} = 2 \cdot PS1_{ave} \cdot aGR \frac{PS1_{ave} \cdot R1}{4 \cdot PP1 \cdot R2} = 1 \cdot PS1_{ave} \cdot R1 \cdot 4 \cdot PP1 \cdot R2.
\]  

(16)

It is assumed that \( PS1_{ave} = PS1_{1/2} \) and that the interference will only happen when both mixing signals S1 and S2 bear “one”; hence, the noise power is reduced by 1/4. R1 and R2 are the two R functions for the two procedures. Equation (16) is the first analytical formula to describe the interference induced noise of a signal at the output of the WBC. Then, it can be used to calculate the induced penalty (interrelated penalty) in [24]

\[
\delta_{int} = 10 \log \left( 1 - r^2_{int} \cdot Q^2 \right).
\]  

(17)

Evidently, when one interferer is concerned, (17) overestimates the penalty [27]. Note that only with the assistance of the analytical model can the different crosstalk terms be assessed.

**GM Penalty:** This penalty is related to the fluctuations of PFk_1 and PFk_0 due to the modulation of the gain by the other channels. These crosstalk effects are due the fact that the gain experienced by PF1 is modulated by the aggregate power inserted in the SOA during each symbol of PF1. This means that to evaluate the GM effect individually, one has to assume that the modulated gain will create a discrete number of sub-levels on the power of “1” of PF1. To calculate the penalty due to GM, the variance of these discrete levels has to be calculated and the noise treated as an RIN, and hence, \( \sigma^2_{Gi} = \langle PF1^2 \rangle - \langle PF1 \rangle^2 \), where

\[
\langle PF1 \rangle^2 = \left( PS1_1 \cdot PP1 \sum_{j=0}^{\zeta-1} \left( \begin{array}{c} (\zeta-1)! \\ j!(\zeta-1-j)! \end{array} \right) \right)^2 \times a^3 \left( \frac{\zeta}{\zeta + \frac{\zeta}{ER}} \right)^{-2.4} \frac{R}{2^{\zeta-1}}
\]  

(18)
For the calculation of the GM induced penalty, the inclusion of both $\sigma_{G1}$ and $\sigma_{G0}$ may be significant. If the number of discrete levels is large enough, the distribution can be assumed to be Gaussian. Intuitively, we can argue that for a large number of channels, the effects of a GM act upon the signal more as a saturation effect that limits the ER, rather than a fluctuation effect. The $Q$ parameter in the case of the optimum threshold is given by [24]

$$Q = \frac{I_{11}}{\sqrt{\sigma_{sp}^2 + \sigma_{s}^2 + \sigma_{G1}^2 + \sigma_{G0}^2}}$$  \hspace{1cm} (19)$$

where $I_{11} = R_D \cdot PF_{11}$, and $I_{10} = R_D \cdot PF_{10} = R_D \cdot PF_{11}/ER \ll R_D \cdot PF_{11}$. Hence, $I_{11} - I_{10} = I_{11}$. Index $T$ indicates the thermal noise and the $s$ shot noise, as in [24]. Also, $\sigma_{G1} = r_1 R_D \cdot PF_{11}$ and $\sigma_{G0} = r_0 R_D \cdot PF_{11} = 2r_0 R_D \cdot PF_{11}$. The receiver sensitivity $P_{rec}$ is the average power in the receiver, for which $Q$ of channels, the effects of a GM act upon the signal more as given by [24]. Also, $\delta_{GM}$ indicates the thermal noise and the shot noise, as

$$\delta_{GM} = 10 \log \left(1 - \frac{Q r_0}{ER}\right)^2 - r_1^2 Q^2.$$

This is the penalty induced by the GM (GM related). Based on the calculations, $\sigma_{G0}$ has a very low value and can be neglected. One has to substitute the values of noise like in (18) in order to obtain $\delta_{GM}$.

**OSNR Penalty:** OSNR related penalty is due to the ASE added to the signal after the converter. The receiver sensitivity is now given by the summation of the signal power and spontaneous emission:

$$P_{rec} = P_{ave} = \frac{PF_{11} + P_{sp}}{2} = \frac{PF_{11}}{2} \left(1 + \frac{\Delta \nu_{opt}}{2 OSNR \Delta \nu_1}\right) = \frac{PF_{11}}{2} \cdot x$$

where $\Delta \nu_{opt}$ is the filter bandwidth after the amplifier, and $P_{sp}$ is the spontaneous emission noise power that enters the receiver. Evidently, $P_{sp} = \Delta \nu_{opt} S_0$ is given by (A.5) [24]. It stems that OSNR $= P_{ave}/S_0 \Delta \nu_1$. The $Q$ factor is now given by

$$Q = \frac{I_{11}}{\sqrt{\sigma_{sp}^2 + \sigma_{s}^2 + \sigma_{G1}^2 + \sigma_{G0}^2}}.$$

The current noise consists of fluctuations due to the shot noise, the thermal noise and the ASE noise. The ASE-induced current noise has its origin in the beating of the signal electric field with the spontaneous emission noise ($\sigma_{s\rightarrow sp}$) but also of the beating of the spontaneous emission with itself ($\sigma_{sp\rightarrow sp}$) [24]. If thermal noise is ignored, the remaining terms are given by [24]

$$\sigma_{s}^2 = 2q R_D (P_1 + P_{sp}) \Delta f$$

$$\sigma_{sp\rightarrow sp}^2 = 4 R_D^2 P s_i \Delta f$$

$$\sigma_{sp\rightarrow sp}^2 = 4 R_D^2 P s_i \Delta f \Delta \nu_{opt}.$$

If the receiver sensitivity is not affected by thermal noise, the back-to-back sensitivity is given by [24]:

$$P_{rec}^\prime = \frac{(q \Delta f / R_D) Q^2}{2}. \quad \text{The penalty } \delta_{OSNR} \text{ equals } P_{rec}^\prime / P_{rec}. \quad \text{For an NRZ system at } B \text{ Gbps, the bandwidth of the receiver has to be chosen appropriately}.$$

This is dictated mainly by the bandwidth of the filters. For optimum performance, $\Delta \nu_{opt} = B$ and $\Delta f = B/2$ should be used to obtain the OSNR related penalty

$$\delta_{OSNR} = 10 \log \left(\frac{1}{x^2} - \frac{2Q}{x^2 OSNR} \sqrt{\frac{B^2}{2 \cdot 12.5^2}} - \frac{2Q^2}{x^2 OSNR 2 \cdot 12.5} \right). \quad (22)$$

**Overall Penalty:** In order to calculate the overall penalty, the $Q$ factor is estimated for a signal that suffers from all the above effects when $\sigma_{G0} = 0$. The reference receiver only suffers from the shot noise. The $Q$ factor can be calculated from (23), shown at the bottom of the next page, where

$$\kappa = \frac{(ER_{out} - 1)}{ER_{out}} \quad \chi = \frac{(ER_{out} + 1)}{ER_{out}} \quad \beta = \frac{ER (ER + 1)}{(ER - 1)^2} \quad \psi = 1 + \frac{\Delta \nu_{opt}}{OSNR \Delta \nu_1}.$$

If no assumptions are made regarding the receiver, the equation has to be numerically calculated. Following the procedure as before, i.e., the receiver sensitivity is not affected by thermal noise, the penalty is analytically calculated from

$$\delta = \frac{k^2 \beta}{\chi \psi} \left[1 - \frac{Q^2}{k^2 OSNR \Delta \nu_1} \frac{2Q \sqrt{\Delta f \Delta \nu_{opt}}}{\kappa} - \frac{Q^2}{k^2 \nu_{opt}} \frac{Q^2 \nu_{opt}^2}{k^2 \nu_{int}^2}\right]. \quad (24)$$

**IV. Experimental Setup**

The experimental setup of Fig. 4 was used to investigate the WBC. In this setup, the pump source is a distributed feedback (DFB) laser, denoted as Laser 1 ($\lambda_1 = 1558.14$ nm). A second laser (Laser 2) is used as a signal at $\lambda_2 = 1556.84$ nm. The spectrum is observed on an optical spectrum analyzer (OSA).
Fig. 4. Experimental setup for the evaluation of WBCs.

with a 0.1-nm resolution. The polarization was manually adjusted for optimum performance of the SOA. The amplifier used here has a multiquantum well (MQW) active medium with a 30-dB unsaturated gain at 200 mA. Unless otherwise stated, the operation of the system was as follows.

The two lasers are modulated simultaneously by two different channels at 10 Gb/s. In order to decorrelate the signals, a fiber delay line is used after Laser 2. The two signals are coupled with two CW pumps and mixed into the SOA. In some experiments though, only one pump is used.

V. TWO-CHANNEL WBC

In order to investigate the performance of the WBC, both the analytical and the numerical models were deployed and compared. The analytical model was used to investigate the power levels and fluctuations and the penalties induced from features like the ER (15), the OSNR (22), the interference (17) and the GM (21), and the overall penalty (24). Also, the numerical model was used to calculate the penalty induced from the ER (15), the OSNR (22), the crosstalk penalty (21), and the overall penalty (23) and compare the results. The scope of this section is twofold: to investigate the performance of the WBC and to validate the analytical model.

A. Signal Power

The input signal power (PS1 or PS2) is a significant feature for the optimization of the WBC. The numerical model has been used to evaluate the performance of the WBC at 10 Gb/s. The frequency (wavelength) spacing between the signals and the pump are \( \Delta \omega = 125 \text{ GHz} \) (\( \Delta \lambda 1 \approx 1 \text{ nm} \)) and \( \Delta \omega 1 = 250 \text{ GHz} \) (\( \Delta \lambda 2 \approx 2 \text{ nm} \)). The input power of \( P1 \) is \( P1 = -0.5 \text{ dBm} \). The input ER is 10.5 dB, and the input OSNR is 58 dB. For the analytical model, \( R1 \) (for channel S1) is \( -29.5 \text{ dB} \) and \( R2 \) (for channel S2) is \( -31 \text{ dB} \), both independent of power. In Fig. 5, the ER and OSNR of F1 are calculated with the two models.

The analytical model is valid for maximum PS1 and PS2 of \( -10 \text{ dBm} \) (see Appendix). For these values, the two models present a very good agreement. For a higher signal power, ER seems to be more enhanced in the results of the numerical model. This discrepancy is mainly due to the accuracy of the analytical model. The OSNR is not affected by this discrepancy.

Fig. 6(a) shows the constituent penalties, as calculated by the analytical model, versus PS1. As expected, the ER penalty follows the trend of the ER output. The same is true for the OSNR-related penalty. As the OSNR increases, the penalty is reduced. The interference related penalty (inter in Fig. 6) may lead to a serious deterioration of the converted signal quality. Nevertheless, for a small number of interferers, the penalty is overestimated. The interference starts becoming important as

\[
Q = \frac{\alpha \Delta \text{signal}}{y} \\
\sqrt{\frac{q^2_{\text{OSNR}} \Delta f}{R_D} y + 2 \frac{P^2_{\text{noise}}}{y^2 \text{OSNR}} \frac{\Delta f}{\Delta \nu^2} + \frac{P^2_{\text{noise}}}{y^2 \text{OSNR}} \left( \frac{\Delta \nu_{\text{opt}} \Delta f}{\Delta \nu^2} + \frac{4 \sigma_T^2}{R^2} \frac{P^2_{\text{RIN,1}}}{y^2} \right) + \frac{P^2_{\text{noise}}}{y^2 \text{OSNR}} \left( \frac{\Delta \nu_{\text{opt}} \Delta f}{\Delta \nu^2} + \frac{4 \sigma_T^2}{R^2} \frac{P^2_{\text{RIN,0}}}{y^2} \right)}
\]

(23)
the interferer becomes of the order of the product itself. The GM-induced penalty [GM in Fig. 6(a)] worsens as the power of the signals grow for a specific pump power. With the increasing signal power, the product becomes indifferent to the power variations, but at the same time the power of the signals cause the GM to increase.

In Fig. 6(b), the overall penalty has been calculated. There are two cases that have been investigated with the analytical model. For the first case (Analytical I), the penalty is calculated by using (13). In the second case (Analytical II), the two channels are assumed to be modulated with the same pattern. This means that the ERout is given by (14), but no GM-related effects are considered. The penalties are similar for very low powers for the two cases as this is related mainly to the OSNR. For higher PS1 powers, the GM-, inter-, and ER-related penalties are enhanced. The optimum signal power $PS_{1,\text{opt}}$, which corresponds to a minimum penalty, is $-12.5$ dBm, and the corresponding pump–signal ratio is approximately $12$ dB for all cases.

The numerically calculated penalty curve has the same trend as the analytical ones, but for lower powers, the calculated penalty is higher due to the numerical penalty calculation process. For high input power, the penalty is attributed to crosstalk effects like interference and GM. It is the crosstalk-related penalty that accounts for the discrepancy here. The GM is overestimated by the analytical model because in reality, the effect is smoothened by the limited carrier lifetime. This is also supported by the fact that the numerical curve lies between the two analytical ones. Also, there are effects that are enhanced by increasing the signal power that are not accounted for by the analysis, like interferers on the pump and the initial signal that affect the signal indirectly.

As far as the second channel (F2) is concerned, the same results are calculated. For the implementation of the analytical model and for the specific channel spacing, no interference is considered on F2. The analytical penalties are plotted in Fig. 6(c). In Fig. 6(d), the analytical results for cases I and II are plotted together with the penalties calculated by the numerical model. The optimized operation here is for a slightly lower signal–pump ratio than in the F1 case, due to the poorer OSNR of F2. Nevertheless, the minimum penalty is approximately the same with F1. This implies that S2 must be of a higher input power, so that F2 suffers the same penalty as F1. This suggests that a preemphasis must be applied for all the channels in a band in order to achieve uniform performance.

In conclusion, as far as PS1 and PS2 are concerned, the optimum performance is achieved for signal powers lower than the single wavelength converter case. This is especially true for the case of a two-signal wavelength converter; an input signal–pump power ratio of $-12$ dB must be used. In [5], the experimental investigation of the effect led to a signal to pump ratio of $15$ dB. In the one channel case, the corresponding ratio is $-9$ dB [20]. Also, a preemphasis technique can be applied to ensure similar performance for the two channels.

B. Pump Power

Another important parameter for the performance of the WBC is the pump power. Numerical and analytical models are used to simulate the penalties as in the previous section. The input signals have the same features (ER, OSNR, etc.). The average input signal powers PS1 and PS2 are equal to $-8$ dBm, while all the other parameters are the same. The analytical model is valid for a PP1 up to $-1$ dBm.

The ER and the OSNR of the output F1 are plotted against the pump power PP1 (Fig. 7). The ER value increases as a function of pump power and is even negative for PP1 less than the signal power ($< -8$ dBm). The OSNR value increases for increasing PP1, but the numerically calculated one is saturated. The effects are translated into a penalty [Fig. 8(a)], and the constituent penalty terms are all reduced with the increase of optical pump power. The GM on F1 is reduced by increasing the pump power. Also, the interference from undesirable products drops, such as PF1 increases. It is shown that for optimum performance, a sufficiently high pump power should be inserted into the SOA [Fig. 8(b)].

For F2, the simulations are performed as previously and the equivalent trends are acquired [Fig. 8(c)]. The numerically calculated penalty is worse than the analytically calculated one [Fig. 8(d)]. This is attributed mainly to the OSNR as well as the crosstalk effects that are not accounted for in the analytical model.

In conclusion, the high pump powers are quite beneficial for a FWM WBC when deployed in a system that adjusts the pump power. However, this is not always possible, especially in a dynamically reconfigurable network.
C. Bit Rate

Another important parameter that affects the system’s design is the bit-rate. A FWM is expected to be beneficial with respect to other techniques, regarding this aspect. The WBCs though are expected to perform under saturation, especially when a large number of channels are converted. The numerical model can be applied to investigate this effect. To compare the induced penalties between signals of different bit rates, the bandwidth of the receiver needs to change.

In (24), the OSNR induced penalty as a function of the bit rate B and OSNR is shown. If the bit rate of a system is increased four times, the OSNR must also be increased four times (i.e., by 6 dB), to achieve the same penalty value. Hence, a 160 Gb/s signal must have an OSNR value of 12 dB higher than that of the 10 Gb/s signal to suffer the same penalty. This means that even if it is assumed that the output power of the 160-Gb/s signal is not degraded with respect to the 10-Gb/s case, the OSNR is the same, and the penalty is worse because a wider optical filter is needed that allows more ASE to enter the receiver. For instance, if a 10-Gb/s FWM product has OSNR$_{10}$ = 20 dB, an acceptable penalty would be induced, assuming that there are no other effects. This penalty would be achieved for a 40 Gb/s product only if it has OSNR$_{40}$ = 26 dB, and for 160 Gb/s, an OSNR$_{160}$ = 32 dB would be required. However, an OSNR value of more than 25 dB is difficult to be achieved, even when high pump and signal powers are used. This consideration makes the 160-Gb/s bit rate almost prohibitive for this WBC.

Nevertheless, the issue can be resolved by utilizing a 160-Gb/s RZ format, rather than NRZ format. Then, the signal can be demultiplexed optically to 10 Gb/s before detection.

In the above discussion, the waveform distortion has been neglected. Here, the numerical model is used to investigate the penalties induced by the WBC and are related to the ER and crosstalk for different bit rates. Three cases are investigated are 10, 40, and 160 Gb/s for NRZ signals similar to the ones described before. $\Delta \lambda 1$ is 1.8 nm, $\Delta \lambda 2$ = 1.61 nm, and P$P_{1}$ = 1 dBm. The ER and crosstalk related penalties are calculated with respect to the signal power and are shown in Fig. 9.

The ER related penalty is associated with the average PF$_{1}$ and PF$_{16}$; hence, the carrier lifetime limited amplification has an effect on the performance. For high input powers, the pattern effects are worst for the 160-Gb/s case and consequently affect the ER of the product. Meanwhile, the crosstalk related penalty appears less for the 40-Gb/s signal and even lower for the 160-Gb/s signal. This is attributed to the smoothing of the GM. Nevertheless, these phenomena are more likely to take place when long sequences of 1 or 0 appear in a 160-Gb/s data sequence. For that reason, the results are of qualitative value, unless longer pattern sequences are deployed.

The above allows important conclusions to be drawn for a node design. In such a system, the constituent effects may be optimized, as for the example of the OSNR or the ER, by the use of output 2R regenerators.

D. Wavelength Dependent Performance of a WBC

The wavelength separation between the pump and the signals is a very important parameter. Of equal importance is the wavelength placement on the SOA gain curve. In Fig. 10, the numerically calculated conversion efficiency (H) is plotted. The pump is placed on the peak wavelength and the signals are moved towards other wavelengths. P$P_{1}$ is 0 dBm and the average PS$1 = PS2 = -10$ dBm. The numerical results are compared with the experimental ones, which was derived utilizing the aforementioned experimental setup.

The simulations have been repeated for different wavelength positions, and the overall penalty was calculated for two cases: when the $\Delta \lambda$ between S1 and S2 is 1.24 and 2.22 nm. In Fig. 11, the penalties versus the wavelength position of the pump are plotted with respect to the gain peak. The wavelength positioning has a small effect on the ER and, thus, to the calculated penalty. The OSNR is enhanced when the pump is placed at the peak of the SOA gain. When S1 or S2 are placed close to the peak gain wavelength, the modulation is worse and affects the performance of the signals. So it is preferable for the initial signals to be placed far from the wavelength peak.

E. Experimental Results for the Two-Signal WBC

The experimental configuration used is illustrated in Fig. 4; here, only one pump is used. The output spectrum for the
representative input powers are shown in Fig. 12(a). By incorporating a preemphasis, the OSNR of the two products is the same. The scope of the experiment was not to directly validate the models but to show that the design rules are quite valid.

An eye diagram of the product F1 is also illustrated in Fig. 12(a). The histograms show that there are two sublevels for the level of “one,” both resembling a Gaussian characteristic due to the ASE noise. By setting $PP1 = 1$ dBm, and for various input signal powers, the BER penalty of F1 with respect to PS1 is measured and plotted in Fig. 12(b). The measured penalty is higher than the one could expect (Fig. 6), but the curve presents a similar trend, and the minimum penalty is approximately at a signal–pump ratio of $-11$ dB. The corresponding penalty value is worse due to the measurement errors, mainly due to the noise added by the preamplified receiver and the bandwidth of the optical filter. Nevertheless, the experimental verification is indicative for the numerical model validation.

VI. Dual-Pump Configuration for WBCs with Two Channels

The dual pump FWM was suggested as a means for the wide tunability for one-signal configurations, and it was first utilized for a WBC in [15]. A dual pump configuration with two parallel pumps has been simulated using the numerical model with $\Delta \omega = 250$ GHz, $PP1 = PP2 = 0$ dBm, and $PS1 = PS2 = -10$ dBm. Furthermore, the analytical model was applied to calculate the OSNR of F1 and F2 as in [20]. The results are shown in the Fig. 13 together with the experimental ones. The gain profile of the SOA that affects both the output power and the ASE governs the results in the longer wavelength region. Nevertheless, the OSNR is high and uniform for a wide range of wavelengths.

Tunable input/tunable output WBCs based on FWM are not suitable due to the large number of channels that must remain unused, as channels cannot be allowed to convert to the same wavelength or any close wavelength. In some configurations [28], a fixed input/tunable output (FITO) WBC is required or a tunable input/fixed output (TIFO).

A. FITO Wavelength Converter

There are three main cases for an FITO, as shown in Fig. 14: a) The fixed pump is on the peak of the gain; b) the fixed pump and signals are close to the band gap; and c) fixed pump and signals are placed towards transparency wavelength. Any other option would benefit one of the two signals. The fixed pump P1 and input signals S1 and S2 perform a down conversion (wavelength).

Assuming a pump power ($PP1 = PP2$) is $0$ dBm and a signal power ($PS1 = PS2$) is $-10$ dBm. A numerically calculated penalty is obtained for various wavelength positions of the second pump (P2), with respect to the peak wavelength and is
plotted in Fig. 15. The dashed line shows the position of P1. The first column presents the results for F1, while the second is for F2 for the three cases. When the P1 is close to the peak, effects like GM are enhanced in comparison with the case wherein both P1 and P2 are quite separated from the peak gain. The OSNR is the main feature that would dictate the overall penalty over the wavelength range. Between the down conversion case [see Fig. 15(c) and (d)] and the up conversion case [see Fig. 15(e) and (f)], there is a main difference as far as OSNR is concerned due to the interplay between the wavelength dependence of the gain and the wavelength dependence of the ASE. Also, in the case of parallel pumps, there is a second mechanism that depends on the conversion wavelength separation [20]. All the above make the down conversion case, where if P1 is close to the band gap, it is the most efficient one.

B. TIFO Wavelength Converter

For the TIFO, we assume that the fixed pump P2 is placed at one of the three following wavelengths (Fig. 16): 1) at the peak wavelength; 2) towards shorter wavelengths; and 3) close to the band gap.

Again, the numerical model has been deployed to calculate the penalty for each case. The signal and pump parameters are the same as above. The penalty is plotted for both F1A (first column) and F2A (second column) for the four cases, together in Fig. 17. The OSNR-, ER-, and crosstalk-related penalty are indicated. The performance is dictated mainly by the OSNR, and follows the same trends as that of the one signal WC [20]. The wavelength dependence of the OSNR is a function of the gain, the ASE, and the conversion efficiency dependence on the wavelength. The combination of these is reflected in the results of the TIFO with 2 dB of OSNR-penalty variation among the different configurations. The crosstalk-related penalty is worse when P1, S1, and S2 are close to the peak gain, and it varies by 1 dB among the different configurations. In this case, the best performance is acquired when the P2 is placed on shorter wavelengths than that of the gain peak.

C. Experimental Results of WBC With Two Channels

In order to implement the TWBC, the experimental setup (Fig. 4) was used with two pumps (P1 and P2) and input powers PP1 = 0 dBm and PP2 = 0 dBm. The wavelengths of P1 and P2 are at 1555.0 and 1542.97 nm accordingly, and their power is −8 dBm. The two input channels S1 and S2 are at 1556.31 and 1558.31 nm and are modulated at 10 Gb/s. The spectrum at the output of the SOA is shown in Fig. 18(a) for a representative set of powers. The BER measurements for a TWBC were taken and plotted in Fig. 18(b). By removing the P2, the BER measurements were taken for the WBC without further optimization
of the configuration. Then, by removing the S2, the same measurements were taken for the WC for comparison. Evidently, the poor OSNR explains the performance deterioration of the TWBC (penalty of 4.1 dB) with respect to the WBC (penalty 2.1 dB). Although the experiment here is used only for one case of input powers, and it is not sufficient for direct comparison with the set of numerically or analytically calculated results, it is still indicative of the validity of the model. The set of powers used was optimal for the TWBC and was proven by means of BER for a specific input power. However, a direct comparison between the experiment and the models is not always possible as the input signals were amplified in order to compensate for the losses so a special numerical model should have been developed for direct comparison.

VII. Design Rules

In the previous section, the performance of the TWBC was investigated. To deduce useful design rules for node design, the analytical model is deployed. The overall penalty of F1 versus the average signal power is analytically calculated for a TWBC, assuming that no interferers are present and PP1 = PP2 = −0.5 dBm. The results are plotted in Fig. 19, together with the results for the WBC for a direct comparison. The optimum performance for the TWBC is achieved by increasing the PS1 by +3 dB with respect to the WBC. This is necessary so that the OSNR is enhanced by increasing the signal power. The ratio between the pump power and the signal power for optimum performance is reduced from +12 to +9 dB. The same is noted when PP1 = 1 dBm. The performance is enhanced for higher pump powers [Fig. 19(b)].

VIII. Conclusion

In this paper, an evaluation of the WBC and the TWBC based on a FWM in SOAs was made focusing on two-channel configurations. The assessment was performed by means of a numerical model, an analytical model, and an experiment. The numerical model calculates the waveform of the output products while the analytical gives directly the levels and fluctuations of the powers. The analytical model was developed based on the study described in [29]. Novel analytical formulas were developed that calculate the induced penalties due to various effects namely ER deterioration, OSNR reduction, GM, and interference. The analytical model, although limited with respect to the input optical power [20], [29], manages to describe the penalty trends. In order to describe the effects in more detail, the empirical gain model used in this model has to be adjusted for higher input powers. This means that values of γ and δ need to be acquired for the specific power range. Furthermore, the analytical model cannot give any insight on the bit-rate performance or the wavelength dependent effects.

On the other hand, the numerical model explains all the necessary effects well, and it is obvious that the OSNR calculations and the efficiency calculations, which may be sufficient for WC, are not adequate for investigating the WBCs. Dynamic effects like cross modulation and interference have to be taken into consideration. The main design rule that has been deduced is that the ratio of the pump power to the signal power should be approximately 12 dB. This is because the presence of the second channel saturates the gain and the ASE of the SOA, affecting the OSNR but at the same time reduces the ER and enhances the GM. The interplay of these effects reduces the ratio between the pump power and the signal power that is required for optimum performance to 12 dB. This is verified by the experimental results shown in Fig. 12(b). Additionally, effects like bit-rate performance, signal-pump separation, etc., have been studied.

A TWBC was also examined in a dual pump configuration, in which case, more complicated effects take place. The SOA operates under a strong saturation by the two pumps and the two signals and the OSNR of the converted signals deteriorated a lot. The FITO and the TIFO configurations in a down conversion scheme are favored in terms of the OSNR. It is also obvious from the analytic results that for the optimum performance of a TWBC, the ratio of the pump power–signal power has to be reduced to 9 dB so that the saturation suffered by the second pump will be overcome by the signal power. It

\[ \xi = 2 \] is used to include the saturation effects from both pumps.
was also discussed that larger powers give a wider dynamic range, as the system is less prone to cross modulation effects.

In conclusion, the WBCs are very important subsystems for future optical networks [28]. An FWM has been assumed as an interesting technique for waveband conversion. In this paper, a systematic theoretical analysis of such a subsystem is presented and performed. Evidently, the performance is very much dependent on parameters like signal and pump power, bit rate, etc., and optimized configurations have to be sought when deployed in systems [28].

APPENDIX

The model assumes that under specific conditions, the SOA can be modeled as a lumped saturable gain section (G), followed by a lumped third-order nonlinearity section ($\chi^{(3)}$), and an ASE section [20], [28]–[30]. For the sake of generality, we assume three beating waves at different frequencies $E_{p1}$ (at $\omega_{p1}$), $E_{s2}$ (at $\omega_{s2}$), and $E_{s1}$ (at $\omega_{s1}$), the beating of which produces a fourth wave at $E_{F1}$ ($\omega_{p1} + \omega_{s2} - \omega_{s1}$). In the conventional pump probe configuration, $\omega_{p1} = \omega_{s2}$. Then, the output electric field is given by [28]

$$E_{F1} = E_{p1}^2 E_{s1} r(2\omega_{p1} - \omega_{s1})^* \exp(i(2\omega_{p1} - \omega_{s1})t) \quad (A.1)$$

where $r$ is the conversion efficiency function $r(2\omega_{1} - \omega_{2}) \sim \sum m A_m \exp(i\phi_m)/(1 - i\tau_m \Delta \omega)$, where $m$ denotes the different processes that contribute to the FWM, as described in the previous section. The explicit form of $r$ given in [29] depends on the strength of the process the gain and the input powers. The output FWM power $P_{F1}$ due to the beating of a pump (power PP$_1$ and frequency $\omega_{p1}$) and a signal (power PS$_1$ and frequency $\omega_{s1}$) at frequency $2\omega_{p1} - \omega_{s1}$ is expected to be

$$P_{F1} = PP_1 PS_1 G^3 R(2\omega_{p1} - \omega_{s1}) \quad (A.2)$$

where $G$ is the SOA gain which is assumed to be spectrally flat, and $R$ is the relative conversion efficiency function

$$R(2\omega_{p} - \omega_{s}) = |r(2\omega_{1} - \omega_{2})|^2$$. Here, the experimentally determined relation for $G$ is used [29].

$$G = bP^{-\gamma} \quad (A.3)$$

where $P$ is the total input power. We will assume that $R$ is independent of the power to investigate the individual effects that dictate the FWM performance of an SOA. It is noted that

$$P_{\text{out}}^{\text{dB}} = 2P_1^{\text{dB}} + P_2^{\text{dB}} + 3G^{\text{dB}} + R(2\omega_{p} - \omega_{s})^{\text{dB}} \quad (A.4)$$

where $R^{\text{dB}} = X^{\text{dB}} + 20 \log_{10}(|\sum_x \varepsilon_x (1-i\Delta \omega_x)/(1-i\Delta \omega_x)|)$, where $X^{\text{dB}}$ can be calculated from [28]. In order to calculate analytically the OSNR characteristics of this model, we assume that the ASE is given by the formula

$$P_{\text{ASE}} = b2P^{-\delta} \quad (A.5)$$

where $P$ is the total input power in the SOA. By means of comparison, the experimentally achieved amplifier gain and ASE the parameters $b$, $\delta$, $\gamma$, $b_2$, and $\delta$ were obtained in [20]. These parameters that gave a good matching for a power spanning from $-15$ to $0$ dBm are $b_1$, $b_2 = 0.88$ and $\gamma = 0.8$ for the gain, while $b_2 = 0.0023$ and $\delta = 0.76$ for the ASE [20].

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