Waveband Assignment in bi-directional Ring Networks

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Abstract: In this paper, we give an integer linear programming formulation and present a heuristic algorithm to minimize the worst-case tuning ROADM range in a bi-directional ring network to support all-to-all traffic.

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1. Introduction

Reconfigurability plays an important role in wavelength-division multiplexing (WDM) through the use of reconfigurable optical add-drop multiplexers (ROADMs). ROADMs and tunable optical transceivers are potential sources of cost minimization. A limited ROADM (L-ROADM) can drop/add only a subset of (contiguous) wavelengths from a full spectrum. L-ROADMs reduce the tunability range of the OADMs in each node of the network, which in turn decreases the number of required switches to add/drop traffic or to allow traffic to pass through without the need for an expensive optical-electrical-optical conversion. On the other hand, a fully reconfigurable OADM, which can add/drop any wavelength, is called an F-ROADM. Since L-ROADMs have smaller tunability band size than F-ROADMs, they are less expensive than F-ROADMs and can reduce the network cost. ROADMs with tunable optical transceivers constrain wavelength assignment algorithms for lightpaths through the wavelength termination constraint [3]. A lightpath can be established between two nodes on wavelength λ\(_i\) only if λ\(_i\) can be added/dropped at both the nodes.

In this work, we consider a bi-directional ring network in which we would like to support any subset of all-to-all traffic, and obtain results on the band sizes of the L-ROADMs. Unlike [2], we assume that the bands of different L-ROADMs may be different. Our general goal in the paper is to minimize the band size of the L-ROADMs and to maximize the number of L-ROADMs. We develop an integer linear programming (ILP) formulation for minimizing the band size when all ROADMs are L-ROADMs and also present a heuristic algorithm for this problem that runs efficiently for large network sizes.

2. Network and Traffic Models, Definitions, and Problem Statement

We consider a bi-directional, two-fiber ring network with \(N\) nodes numbered from 1 to \(N\), each equipped with \(N - 1\) transceivers. The ring topology is represented by a graph \(G = (\mathcal{V}, \mathcal{E})\) where \(\mathcal{V}\) is the set of nodes and \(\mathcal{E}\) is the set of edges or links. A connection between nodes \(i\) and \(j\) includes two lightpaths, one from \(i\) to \(j\) and another from \(j\) to \(i\) along the shortest path (fewest hops). We consider all-to-all traffic \(T = \{(i, j) | i \neq j, i, j \in \mathcal{V}\}\). Let \(W\) be the number of wavelengths needed to support the traffic. Each ring node has either an F-ROADM, which can add/drop any of the \(W\) wavelengths, or an L-ROADM. Let the contiguous set of wavelengths (which we call as waveband) that can be added/dropped by the L-ROADM at node \(i\) be denoted by \(B_i\), i.e., any wavelengths within \(B_i\) can be added/dropped at node \(i\), and the wavelengths within \(B_i\) are contiguous in the spectrum. We then define the band size or range of the L-ROADM at node \(i\) to be \(R_i = |B_i|\). Note that we allow the wavebands to be different for different nodes. As described earlier, L-ROADMs introduce a wavelength termination constraint. We say that \(T\) is supportable if every connection in \(T\) can be established using one of the \(W\) wavelengths and respecting the wavelength continuity and wavelength termination constraints. Note that if \(T\) is supportable, then any subset of \(T\) is also supportable. The goal of our work is to minimize the waveband sizes so that a given traffic set is supportable. While there are many ways to pose this general problem, in this paper, we choose to solve the following problem: Given \(T\) and \(W\), determine the band sizes of the ROADM such that the worst-case band size \(R = \max_{i \in \mathcal{V}} R_i\) is minimized and \(T\) is supportable.

3. Results

Let a traffic set \(T\) consists of a single traffic \(T = \{(i, j) | i, j \in \mathcal{V}, i \neq j, \text{dist}(i, j) \leq (N - 1)/2\}\) where \(\text{dist}(i, j)\) is the number of hops from \(i\) to \(j\) along the shortest path in the ring. Our goal is to derive the band sizes for each ROADM so that the worst-case band size is minimized and all-to-all traffic is supportable. With shortest-path routing, the number of wavelengths necessary and sufficient to support all-to-all traffic is \(W_T = (N^2 - 1)/8\) for odd \(N\) [1]. To illustrate the problem, consider a 7-node ring. We need \(W = 6\) wavelengths, and the naive solution is to require all nodes to be F-ROADMs. Now, consider the assignment of wavelengths to connections shown in Table 1.\(^1\):

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\(^1\)The other connections are made using the same six wavelengths in the other direction.
From this, we see that the ROADM bandsizes are: 3 for node 1 ($\lambda_1$ to $\lambda_3$), 5 for node 2 ($\lambda_1$ to $\lambda_3$), 5 for node 3 ($\lambda_2$ to $\lambda_0$), 4 for node 4 ($\lambda_3$ to $\lambda_0$), 3 for node 5 ($\lambda_1$ to $\lambda_3$), 4 for node 6 ($\lambda_2$ to $\lambda_3$) and 4 for node 7 ($\lambda_3$ to $\lambda_0$). Thus, none of the ROADMs need to be F-ROADMs, and the worst-case band size is 5 for node 2 and 3.

Now, given an $N$-node network, we are interested in determining the worst-case band size of the ROADMs. We believe that this problem may be NP-hard. In the following, we present a simple integer linear programming (ILP) formulation to solve the problem. Even though this is an off-line network design problem, solving the ILP takes an extremely long time (days to weeks) even when $N$ becomes moderately large (about 20), and accordingly, we also present a simple heuristic algorithm.

### 3.1 An ILP Formulation

Recall that $N$ is the number of nodes, and $R_i$ is the band size of node $i$.

- Given: $L^s(i, j) = 1$, if connection $(i, j)$ crosses link $s$; else $L^s(i, j) = 0$. This is the routing.
- Define: $C^k(i, j) = 1$, if wavelength $\lambda_k$ is assigned to connection $(i, j)$; else $C^k(i, j) = 0$.

We further define the following variables:

- $\lambda_{\text{min}}^i = \min \{ k | C^k(i, j) = 1 \}$ is the wavelength with the smallest index used by node $i$.
- $\lambda_{\text{max}}^i = \max \{ k | C^k(i, j) = 1 \}$ is the wavelength with the largest index used by node $i$.

With these definitions, we have the band-size of node $i$

$$R_i = \lambda_{\text{max}}^i - \lambda_{\text{min}}^i + 1.$$

The problem can now be posed as the following optimization:

$$\min_i R_i$$

subject to the following constraints:

- $\sum_k C^k(i, j) = 1 \ \forall i, j$. This constraint ensures that there is only one wavelength used for the connection $(i, j)$.
- $\sum_{(i,j)} \sum_k L^s(i, j) C^k(i, j) \leq 1 \ \forall s$. This constraint ensures that all connections crossing the same link $s$ are assigned different wavelengths.

### 3.2 A Heuristic Algorithm

We now present a wavelength assignment heuristic to minimize the worst-case band size. Recall that the analysis done in one direction can be applied to the other direction. Define $s = (N - 1)/2$; this is the number of hops on the longest connection. For $N$ odd, sequentially starting with 1 and ending at node $N$, establish lightpaths so that each arbitrary node $i$ connects to $(i + 1, i + 2, \ldots, i + s \mod N)$. For simplicity, let us define the $j$th connection from an arbitrary node to be $H_j$. For odd $i$, let $i_{op}$ and $i_{en}$ be next odd node and previous odd node, respectively. Similarly, for $i$ even, let $i_{en}$ be the next even node and $i_{op}$ be the previous even node. Our algorithm assigns first-fit wavelengths to connections ordered in a particular manner. First, we assign wavelengths to node 1’s connections ($H_1, H_2, \ldots, H_s$), i.e., connections to 2, 3, 4 in a 7-node ring; and then to node 2’s connections ($H_s, H_1, H_2, \ldots, H_{s-1}$), i.e., connections to nodes 5, 3, 4. For the remaining nodes, connections for $i_{on}$ start at ($H_{i_{on}+s-2 \mod (s)}$); this is done by shifting the start of the connections of $i_{op}$ by $(s - 2)$. Similarly, $i_{en}$’s connections start at ($H_{i_{en}+s-2 \mod (s)}$). The results of applying this heuristic assignment to a 7-node ring is given in Table 1.

The connections in Table 1 are ordered in the same way that the algorithm considers the connections.

<table>
<thead>
<tr>
<th>Wavelengths</th>
<th>$N_1$ to</th>
<th>$N_2$ to</th>
<th>$N_3$ to</th>
<th>$N_4$ to</th>
<th>$N_5$ to</th>
<th>$N_6$ to</th>
<th>$N_7$ to</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Wavelength assignment for a 7-node ring.
3.3 Performance Analysis

Here, we evaluate the performance of our wavelength assignment algorithm. Fig. 1(a) shows the comparison of the algorithm's results to optimal results (obtained by solving the ILP using CPLEX) for both uni- [4] [5] and bi-directional ring networks. For all of the networks that we have considered, heuristic results are very close to the optimal results. For the 15-node network, our algorithm for the uni-directional ring uses only 65.7% of the full range, whereas the optimal value is 61.9%. For the bi-directional ring network, the heuristic uses 82.1% and the optimal value is 78.5% of the full range, $W_F$. The heuristic provides a saving of 34.3% of the full range for the uni-directional and 18% for the bi-directional ring network. This reduces the total cost of the network.

Fig.1(b) shows the savings in ROADM tuning range over the full range of wavelengths for both uni- and bi-directional rings using our algorithms. As can be seen, uni-directional rings provide savings of between 20-40% and bi-directional rings provide savings of 10-20%.

<table>
<thead>
<tr>
<th>No. of Nodes</th>
<th>Full Range Band size</th>
<th>Heuristic Results Band size</th>
<th>Optimal Results Band size</th>
<th>Band size as a % of Full range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uni</td>
<td>Bi</td>
<td>Uni</td>
<td>Bi</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>3</td>
<td>8</td>
<td>3</td>
</tr>
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<td>6</td>
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<td>5</td>
</tr>
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<td>9</td>
<td>36</td>
<td>10</td>
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</tr>
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<td>21</td>
<td>52</td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td>105</td>
<td>28</td>
<td>69</td>
<td>23</td>
</tr>
</tbody>
</table>

Fig. 1. Performance Comparison(a)Optimal and heuristic results for both uni/bi-directional ring networks, and (b) Band size saving for various values of N of uni/ bi-directional ring networks.

4. Concluding Remarks

We have considered the problem of wavelength assignment to all-to-all traffic in bi-directional ring networks with the objective of minimizing the tuning range of ROADMs. We formulated the problem as an integer-linear program and also presented an efficient heuristic algorithm. Results suggest that a saving of 10 to 20% is achievable in tuning range for this type of traffic.

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References

3. O. Turkcu and S. Subramaniam, Blocking in Reconfigurability Optical Networks, in Proc. INFOCOM '07.